Accepted Manuscript

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PII: S0893-9659(17)30095-2
DOI: http://dx.doi.org/10.1016/j.aml.2017.03.015
Reference: AML 5213

To appear in: *Applied Mathematics Letters*

Received date : 7 February 2017
Revised date : 24 March 2017
Accepted date : 24 March 2017

Nodal solutions for noncoercive nonlinear Neumann problems with indefinite potential☆

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Abstract

We consider a nonlinear Neumann problem driven by a nonhomogeneous differential operator and an indefinite potential. Using variational methods together with flow invariance arguments, we show that the problem has at least one nodal solution. The result presented in this paper gives an answer to the open question raised by Papageorgiou and Rădulescu [N.S. Papageorgiou, V.D. Rădulescu, Coercive and noncoercive nonlinear Neumann problems with indefinite potential, Forum Math. 28 (2016) 545-571].

Keywords: Nonhomogeneous differential operator; Nodal solution; Variational approach; Gradient flow; Superlinear reaction.
Mathematics Subject Classification: 35J20; 35J60; 35J92; 58E05

1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a C^2-boundary $\partial\Omega$ and let $1 < p < \infty$. In this paper we study the following nonlinear Neumann problem:

$$\left\{ \begin{array}{ll} -\mathrm{div}a(Du(z)) + \beta(z)\,|u(z)|^{p-2}\,u(z) = f(z,u(z)) & \text{in } \Omega, \\ \frac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega, \end{array} \right. \tag{1}$$

where $a : \mathbb{R}^N \to \mathbb{R}^N$ is a continuous, strictly monotone map that satisfies certain regularity conditions. The precise hypotheses on the map are listed in the hypotheses H(a) below. The nonlinearity $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Caratheodory function. Finally, we mention that the potential function $\beta \in L^\infty(\Omega)$ may change sign (indefinite potential) and in the boundary condition n denotes the outward unit normal vector on $\partial\Omega$.

Recently, multiplicity results were proved for equations driven by nonhomogeneous differential operators. We refer to the works of Aizicovici-Papageorgiou-Staicu [1], Papageorgiou-Rocha [23], Papageorgiou-Winkert [24] (Dirichlet problems) and Hu-Papageorgiou [12], Motreanu-Papageorgiou [18], Papageorgiou-Rădulescu [22] (Neumann problems). In the above papers, it is shown that the problem has at least three nontrivial solutions, two of which have constant sign (one positive and the other negative). However, none of the above works provide sign information for the third solution. In this paper, by virtue of the two constant sign solutions obtained in [22, Theorem 4.14], we show the third solution is nodal for problem (1) with superlinear reaction which does not satisfy the AR-condition. Our result gives an answer to the open question raised by Papageorgiou-Rădulescu [22, Remark 4.15]. Our approach is based on variational methods combined with flow invariance arguments. For problems driven by the p-Laplacian, we mention the works of Aizicovici-Papageorgiou-Staicu [2], Gasiński-Papageorgiou [8] and Kyritsi-Papageorgiou [13]. In [2], nodal solutions were obtained for Neumann problems with $\beta(\cdot) = \beta > 0$ and a reaction satisfying the AR-condition. For problems with indefinite potential, we also mention the works of Gasiński-Papageorgiou [9-11], which did not produce nodal soltions.

☆Supported by Science and technology plan foundation of Guangzhou (No. 201607010218) and by Natural Science Foundation of Guangdong Province (No. 04300001).
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