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Research Paper

Model for the prediction of grain density and pressure distribution in hopper-bottom silos

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A model was developed to predict the grain density, pressure distribution and grain mass in hopper-bottom silos. The model consisted of a series of differential equations derived from the force equilibrium on a differential element of grain in the silo. These differential equations govern the relationship between the variable grain density and the stresses in the grain mass. An oedometer was used to measure the bulk density of wheat under various pressure levels. Based on the experimental data, a quadratic equation was proposed to model the relationship between the grain density and the maximum principal stress. The model predicted that grain density, and vertical and lateral pressures in the grain mass increased with the grain depth in the cylindrical portion of the hopper-bottom silo, but decreased with the depth in the hopper. The lateral pressure predicted by the model was greater than that calculated by the Janssen equation for the cylindrical section of the silo. The model predictions of grain mass in silos were compared with the measured values from commercial grain silos at two locations, and differences were found to be less than 1.45%.

1. Introduction

Bulk density is a critical parameter for designing grain storage systems in terms of predicting grain pressures, determining the resistance to airflow, and estimating the grain mass in storage bins. A common assumption is that the bulk density is constant within a silo. However, as a porous material, the bulk density of grain varies with pressure to which it subjected (and thus with the grain depth in the silo). For predicting grain pressure, Haque (2013) pointed out that the assumption of constant bulk density in the commonly used Janssen pressure equation was flawed and resulted in underestimation of loads in storage bins. Haque (2011) showed that considering the effect of increases in bulk density (decreases in void fraction) with grain depth would resulted in higher airflow resistance than that reported by Shedd (1953) for cereal grains. Thompson and Ross (1983) studied the effect of overburden pressure on bulk density of wheat and found a change of 64.2 kg m^{-3} in bulk density over a pressure range of 70–172 kPa, with the largest change at pressure levels below

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Nomenclature

A	Cross-sectional area of the silo (m^2)
A_a	Area of the top surface of differential slice in the hopper (m^2)
A_b	Area of the bottom surface of differential slice in the hopper (m^2)
C	Circumference of the cylindrical silo (m)
g	Acceleration due to gravity (m s^{-2})
H_1	Height of the cylindrical section of silo (m)
H_2	Height of the hopper section of silo (m)
H_c	Height of grain sample in the oedometer (m)
k	Lateral to vertical pressure ratio
n	Total number of grain layers in the silo
n_1	Number of grain layers in the cylindrical section
n_2	Number of grain layers in the hopper section
p_{hmax}	Maximum principal stress at the wall (kPa)
p_{hmin}	Minimum principal stress at the wall (kPa)
p_m	Maximum principal stress (kPa)
p_n	Minimum principal stress (kPa)
p_v	Vertical pressures (kPa)
p_h	Horizontal pressures (kPa)
p_{vc}	Vertical stress in the grain mass along the centreline of the hopper (kPa)
p_{ve}	Vertical stress in the grain mass at the hopper wall (kPa)
p_{hv}	Average vertical stress on a differential slice of grain in the hopper (kPa)
p_0	Applied compression pressure (kPa)
$\bar{p}_v$	Average vertical compression pressure in the oedometer (kPa)
r	Radius of the differential slice in the hopper (m^2)
R	Hydraulic radius of the cylindrical section of silo (m)
R_c	Radius of the sample container of oedometer (m)
S	Area of side surface of differential slice in the hopper (m^2)
V	Volume of differential slice in the hopper (m^3)
μ	Coefficient of wall friction
μ_c	Coefficient of wall friction of the sample container of oedometer
ρ	Compressed bulk density of grain (kg m^{-3})
ρ_0	Bulk density of the top surface layer of grain (kg m^{-3})
φ	Angle of internal friction (°)
α	Half angle of hopper (°)
λ	Intermediate variable

14 kPa. They explained that changes in bulk density at low pressure levels (7–35 kPa) was caused by the rearrangement of grain kernels, whereas, the changes in bulk density at higher pressures (35–172 kPa) were mainly due to deformations of the grain kernels. Similar observations have been made for various bulk materials by other researchers (Moya, Ayuga, Guaita, & Aguado, 2002, 2006; Ramírez, Moya, & Ayuga, 2009). Turner et al. (2016) reviewed several studies on the effect of pressure on bulk density and summarised seven models for the pressure–density relationship. In most studies, the changes in bulk density are related to the "overburden pressure". However, the term "overburden pressure" is somewhat ambiguous. Specifically, it is not clearly defined if the overburden pressure is the vertical pressure in the grain mass or the pressure exerted on the grain surface (the surface applied pressure is used to cause compaction in most experimental studies reported in the literature). Furthermore, in theory the change in volume (bulk density) of a material is directly affected by the hydrostatic pressure (or principal stresses), which counts for both vertical and horizontal stresses (pressures), as well as shear stresses, in the grain mass in a silo. Even if the vertical pressure is the same in two different grain silos, the other stress components may differ depending on the grain and silo properties, such as internal friction angle of grain, grain-silo wall friction, and silo geometry. This means that grain in two different silos could be subjected to different compressions even if the vertical (overburden) pressures are the same in both silos. The concept of overburden pressure becomes even more ambiguous in the hopper region in hopper-bottom silos, where the principal stresses deviated further from the vertical and horizontal directions and grain compaction models based on the vertical stress cannot be applied to the hopper section. The objective of this study was to develop a model to relate the grain bulk density to the principal stresses for both flat and hopper-bottom silos.

2. Model development

2.1. *Density and pressure relationship in hopper bottom silos*

Several researchers proposed mathematical relationships between the bulk density and vertical pressure for grains, as reviewed by Turner et al. (2016). Among these mathematical relationships, polynomials were shown to be capable of capturing the effects of both vertical pressure and moisture content (e.g., Thompson & Ross, 1983). In this study, a second order polynomial was used to correlate the bulk density to the maximum principal stress:

$$\rho = ap_m^2 + bp_m + c \quad (1)$$

where ρ is the bulk density; p_m is the maximum principal stress (pressure); and a, b and c are empirical constants.

The three empirical constants a, b and c were obtained by curve fitting. Specifically, the applied pressure and volume change data were recorded in the compaction tests (described Section 2.2). The principal stress was calculated from the applied pressure and the density was calculated from the volume change. Microsoft Excel was then used to plot the density against the principal stress, and a second order polynomial was used to fit the data to determine the three constants. Tests were conducted for wheat at five moisture levels, generating five sets of data, and each data set resulted in a set of regression constants (a, b and c) for a specific moisture content.