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Robust adaptive tracking control of uncertain slowly switched linear systems

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ABSTRACT

In this paper, robust adaptive tracking control schemes for uncertain switched linear systems subject to disturbances are investigated. The robust adaptive control problem requires the design of both adaptive and switching laws. A novel adaptive law is proposed based on an extended leakage approach, which does not require knowledge of the bounds of the uncertainty set. Two switching laws are developed based on extended dwell time (DT) strategies: (a) mode-dependent dwell time (MDDT); (b) mode–mode-dependent dwell time (MMDDT). MDDT exploits the information of the known reference model for every subsystem, i.e., the dwell time is realized in a subsystem sense. MMDDT is a variant of MDDT that can guarantee stability under faster switching than MDDT, provided that the next subsystem to be switched on is known. The proposed adaptive schemes can achieve global uniform ultimate boundedness for shorter switching intervals than state-of-the-art adaptive approaches based on DT. In addition to global uniform ultimate bounded stability, transient and steady-state performance bounds are derived for the tracking error. The numerical example of a highly maneuverable aircraft technology vehicle is adopted to demonstrate the effectiveness of the proposed adaptive methods.

1. Introduction

Switched systems are an important class of hybrid systems consisting of subsystems with continuous dynamics, called *modes*, and a rule, called *switching law*, to regulate the switching action between the modes. Switched systems appear in a wide range of applications, such as intelligent transportation systems, power electronics, and smart energy systems [1].

To date, productive research has been conducted on switched systems with known parameters, such as stability and stabilization problems [2–7]. This research direction is mainly based on two families of switching laws: dwell time (DT) and average dwell time (ADT) [8]. In DT switching, the switching interval between two consecutive discontinuities of the switching law should be larger than a sufficiently large constant to guarantee the stability of the switched system. In ADT switching, the switching interval between two consecutive discontinuities of the switching law should be sufficiently large in an average sense: this means that very short switching intervals are allowed provided that they are compensated by long ones. Recently, conservativeness[1] of ADT has been further decreased by a new switching strategy proposed in [9]:

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[1] In this work, the term conservativeness is adopted to indicate the lower bound on the length of the switching intervals for which stability can be guaranteed.

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mode-dependent average dwell time (MDADT). The peculiarity of this switching strategy consists in exploiting the information of every mode, such as the exponential rate of the Lyapunov function associated to each mode.

On the other hand, research on uncertain switched systems is not equally mature. As a matter of fact, in real-life problems parametric uncertainty is a ubiquitous condition. This creates additional difficulties when designing control and switching laws. In general, there are two main families of techniques dealing with stabilization of uncertain systems: robust control and adaptive control. It is well recognized that a single robust controller may lead to very conservative performance for a large uncertainty set [10,11]. Therefore, when the uncertainties are polytopic, using a family of robust controllers has been proposed to improve the performance of a single controller [12]. As a complement to robust control, adaptive approaches for non-switched uncertain systems have been investigated to improve the performance of robust approaches over large non-polytopic uncertainties [13–16]. However, adaptive control of uncertain switched systems is more challenging. This is because not only an adaptive law should be developed to estimate the unknown parameters, but also a switching law should be carefully designed to guarantee the stability of the closed-loop system. Recently, some research has been conducted on adaptive tracking control of uncertain switched systems, i.e., switched nonlinear systems [17,18] where adaptive fuzzy approaches are adopted, state-dependent switched systems [19–22] where minimal control synthesis algorithm is used, and time-constraint switched systems [23–26]. However, the following two gaps can be identified in the state of the art on adaptive tracking control for uncertain slowly switched systems: first, the set where the nominal parameters reside should be known *a priori* [23–26]; second, not much attention has been paid to switching laws that exploit the information of each subsystem. While the importance of overcoming the knowledge of the uncertainty set is evident, the need to address less conservativeness switching laws stems from the following research problem: ADT and MDADT switching strategies might cause undesired transient performance of the switched system due to overshoot of the Lyapunov function [1,27]. Therefore, it is relevant to address the following question: can we design an adaptive law and a switching law for uncertain switched linear systems such that the knowledge of the residing space of the parameters is not necessary, and undesired transient behavior of the tracking error due to fast switchings can be avoided?

The main contribution of this paper is twofold. On the one hand, a robust adaptive law with a leakage approach is developed without requiring a priori knowledge of the uncertainty set. On the other hand, two switching laws with shorter switching intervals than dwell time are introduced. In particular, new adaptive tracking control scheme for uncertain switched linear systems is developed based on a mode-dependent dwell time (MDDT) switching law by exploiting the information of every subsystem [28]. Furthermore, to address scenarios for which the next subsystem to be switched on is known, we introduce a new switching scheme: mode–mode-dependent dwell-time (MMDDT). MMDDT is relevant in many applications, such as automobile power train [29], power converters [30], thermostatic control [31], train trajectory planning [32], where the next mode to be switched on is known in advance. Exploiting this information allows even shorter switching intervals than MDDT. Global uniform ultimate stability of the switched system via the proposed robust adaptive tracking control schemes is shown. An upper bound and the ultimate bound characterizing the global uniform ultimate boundedness of the tracking error are also given.

The paper is organized as follows. The problem and some definitions are presented in Section 2. In Section 3, an adaptive law and two switching laws based on mode-dependent dwell time and mode–mode-dependent dwell time are explained. In Section 4, stability results of the closed-loop system are given. In Section 5, a practical example of highly maneuverable aircraft technology is used to illustrate the proposed control schemes. The paper is concluded with Section 6.

Notations: The notations used in this paper are as follows: $\mathbb{R}$ and $\mathbb{N}^+$ represent the set of real numbers and positive natural numbers, respectively. For a symmetric matrix $P > 0$ means P is positive definite. In addition, the superscript T represents the transpose of matrix. The operator $\mathrm{tr}(\cdot)$ represents the trace of a matrix. The notation $\|\cdot\|$ represents the Euclidean norm. The identity matrix with dimension n is denoted with $I_{n\times n}$. The notation $\Omega = \{1, 2, \ldots, N\}$ represents the set of subsystems and N is the number of subsystems.

2. Problem formulation and preliminaries

Consider the uncertain switched linear system described by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + d(t), \quad \sigma(t) \in \Omega \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input, σ is the switching signal, and $d \in \mathbb{R}^n$ is a bounded disturbance with known upper bound $\bar{d}$. We say that a subsystem $p \in \Omega$ is uncertain when the entries of the matrices $A_p \in \mathbb{R}^{n\times n}$ and $B_p \in \mathbb{R}^{n\times m}$ are unknown.

A group of switched reference models representing the desired behavior of each subsystem is given as follows:

$$\dot{x}_{\mathrm{m}}(t) = A_{\mathrm{m}\sigma(t)}x_{\mathrm{m}}(t) + B_{\mathrm{m}\sigma(t)}r(t), \quad \sigma(t) \in \Omega \tag{2}$$

where $x_{\mathrm{m}} \in \mathbb{R}^n$ is the desired state vector, and $r \in \mathbb{R}^m$ is a bounded reference input. The matrices $A_{\mathrm{m}p} \in \mathbb{R}^{n\times n}$ and $B_{\mathrm{m}p} \in \mathbb{R}^{n\times m}$ are known and $A_{\mathrm{m}p}$, $p \in \Omega$, are Hurwitz matrices. The state-feedback mode-dependent control that makes the switched systems behave like the reference models is $u(t) = K^{*T}_{\sigma(t)}(t)x(t) + L^{*}_{\sigma(t)}(t)r(t)$, where $K^*_p \in \mathbb{R}^{n\times m}$ and $L^*_p \in \mathbb{R}^{m\times m}$, $p \in \Omega$, are