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Stability analysis of discrete-time switched systems via Multi-step multiple Lyapunov-like functions[☆]

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ABSTRACT

In this paper, we propose a Multi-step multiple Lyapunov-like functions (MLFs) based approach to prove the stability and asymptotic stability of discrete-time switched systems under pre-given time-dependent switching signals. We start with the definition of Multi-step MLFs, which is a relaxation of traditional MLFs. Then, for a discrete-time switched nonlinear system (SNS) under a given time-dependent switching signal, several sufficient conditions for its stability and asymptotic stability are successively provided by using either Multi-step MLFs or strengthened Multi-step MLFs, respectively. Afterwards, for a discrete-time switched linear system with a special time-dependent switching signal, we derive two less conservative linear matrix inequality based criteria for demonstrating its asymptotic stability. In the end, three examples are given to illustrate the applicability and advantage of our relaxed results.

1. Introduction

Switched systems, as a special class of hybrid dynamical systems [1], are composed of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching sequences between them [2–7]. Actually, switched systems can appropriately model a wide class of systems which exhibit switching features such as physical systems, chemical processes, aircraft control, networked control systems, manufacturing systems, automotive engine control and so on (see [8–13] and references therein).

Among numerous interesting topics of switched systems, stability analysis is a fundamental and significant issue. Specially, for stability analysis of a switched system at its equilibrium point, researchers primarily concern with the following three interesting research directions: the first research direction is to propose sufficient and necessary conditions that can guarantee stability of a switched system under arbitrary switching signals [14–19]; the second research direction is to design switching signals to stabilize a family of individually unstable subsystems [20–23]; and the third research direction is to demonstrate the stability property of a switched system under a given switching signal [7,9,24–32].

In particular, the research direction of analyzing stability of a switched system under a pre-given switching signal has recently received growing attention [7,9,27–31]. As we all know, the existence of a common Lyapunov function (CLF) for all subsystems can guarantee the stability of a switched system under arbitrary switching signals [5]. Thus, the stability of the switched system under any pre-given switching signal can still be obtained via the CLF based method. However, utilizing the existence of a CLF to verify the stability of a switched system under the pre-given switching signal seems too

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conservative. Additionally, the CLF based approach is usually very difficult to apply since for most switched systems, it is difficult to find a CLF in practice even though it exists. Hence, the powerful finite-step Lyapunov function based method has been recently proposed for stability analysis of discrete-time switched linear systems [33–35] and switched nonlinear systems [33,36,37] with state dependent switchings. Moreover, more attention has been alternatively paid to the multiple Lyapunov or Lyapunov-like function (MLF) based approach, which has also been proven to be a powerful and effective method.

The multiple Lyapunov-like function (MLF) based method is firstly proposed in [25] for continuous-time switched linear systems as follows. Under the assumption that the activation time of each subsystem is bounded above and below by some constants, [25] shows that if each subsystem has its own Lyapunov-like function which is positive definite and continuous differentiable and furthermore the value of Lyapunov-like function at the consecutive times (i.e., switching on or off time) is a decreasing sequence, then stability is guaranteed. Later, this result has been extensively generalized for continuous-time switched nonlinear systems in the literature [2,9,24,26–31,38]. For example, [24] demonstrated that when each subsystem possesses a positive definite Lyapunov-like function individually and satisfies the following conditions: (a) each Lyapunov-like function does not increase when its corresponding subsystem is active; (b) each Lyapunov-like function does not increase at its each switching on instant, the stability can be guaranteed; the asymptotic stability can be also guaranteed in [26] and [38] by requiring the sequence of values of the Lyapunov-like function at consecutive switching times to be decreasing and the Lyapunov-like function between these times is bounded by a continuous function; [28] proposed a MLF based theorem by using the idea of evaluating the average value of each Lyapunov-like function during the activation period of each subsystem; [30] constructed a less restrict condition which allows some increase of the values of the Lyapunov-like function at consecutive switching times. Note that the aforementioned MLF based approaches require all the subsystems have a common equilibrium point while [7,29] can be applied to systems whose subsystems may not share a common equilibrium point.

With regard to the discrete-time switched systems, the MLF based method has also been to some extent studied in the literature [2,24,27,39,40] since compared with the CLF based method, the MLF based approach requires less conservative conditions on the Lyapunov-like functions. For example, [27] gives some stability results for discrete-time switched systems by combining the method of MLFs with the comparison principle; [39] achieves linear matrix inequalities based stability criteria for discrete-time switched linear systems based on the MLF based method; [40] studies the global asymptotic stability of the 2D discrete switched systems represented by Roesser models via MLF based approach. However, these existing results on the MLF based method for discrete-time switched systems are somewhat less convenient to check the stability of a switched nonlinear system under a pre-given switching signal in practice. Thus, inspired by the idea of using finite-step Lyapunov functions [37] and the technique of using path-complete graph Lyapunov functions [19,32], we in this paper follow the concept of **T**-step CLF in [41] and propose a Multi-step MLF based method for the discrete-time SNS with a pre-given time-dependent switching signal such that our Multi-step MLF based method can to some extent facilitate the stability analysis of more discrete-time switched systems. Note that [19,32] consider switched linear systems while we in this paper even investigate switched nonlinear systems with time-dependent switching signals; [37] can handle switched nonlinear systems under state-dependent switchings while we here consider systems under time-dependent switching signals.

We start with the definitions of **T**-step Lyapunov-like function and Multi-step MLF. Then, by the existence of Multi-step MLFs, we present two sufficient conditions for a discrete-time SNS to be stable and asymptotically stable under a pre-given switching signal, respectively. Afterwards, we strengthen these two sufficient conditions by additionally comparing the values of the Multi-step MLF at consecutive switching instants. Further, for switched linear systems, we propose two linear matrix inequalities based sufficient conditions, which can be easily used to verify the global asymptotic stability of switched linear systems under certain constrained switching signals. Finally, three examples are given to illustrate the applicability and advantage of our results. Notice that our concept of Multi-step MLFs absorbs the advantages of both the classical MLF in [24] and the finite-step Lyapunov function in [37] such that our Multi-step MLF based method can to some extent facilitate the stability analysis of more discrete-time switched systems under time-dependent switching signals. Moreover, our results can be more easily tested than the existing extension results in the discrete-time counterpart of [28,30,38] in the sense of the convenience for discovering the required MLFs.

The main contribution of this paper can be summarized as follows.

(i) We introduce the definitions of **T**-step Lyapunov-like functions and Multi-step MLFs, which relax the traditional definitions of Lyapunov-like functions and MLF in [24] respectively in sense that the relaxed one decreases after finite steps instead of at each step. Thus, Lyapunov-like functions and MLFs can be regarded as arbitrary-step Lyapunov-like functions and arbitrary-step MLFs respectively, which can be seen from Example 1 in Section 2;

(ii) We prove that the existence of Multi-step MLFs is a sufficient condition for stability of discrete-time SNSs under pre-given time-dependent switching signals. Thus, our method can to some extent facilitate the stability analysis of more discrete-time switched systems, compared with the traditional MLFs based method in [24] and the finite-step Lyapunov function based method in [37], which can be seen from Examples 2 and 4 in Section 5, respectively;

(iii) Two less conservative linear matrix inequalities based sufficient conditions are derived for proving the asymptotic stability of switched linear systems, which both extend previous results in [39] and can be seen from Example 3 in Section 5.