Nonlinear Analysis: Hybrid Systems 27 (2018) 107–124

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Averaging principles for functional stochastic partial differential equations driven by a fractional Brownian motion modulated by two-time-scale Markovian switching processes

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ARTICLE INFO

Article history:
Received 21 April 2016
Accepted 11 August 2017

Keywords:
Averaging principle
Functional stochastic partial differential equation
Fractional Brownian motion
Two-time-scale Markov chain

ABSTRACT

Motivated by applications of hybrid systems, this work considers functional stochastic partial differential equations (FSPDEs) driven by a fractional Brownian motion (fBm) modulated by a two-time-scale Markov chain with a finite state space. Our aim is to obtain an averaging principle for such systems with fast–slow Markov switching processes. Under suitable conditions, it is proved that there is a limit process in which the fast changing "noise" is averaged out and the limit is an average with respect to the stationary measure of the fast-varying processes. The limit process, being substantially simpler than that of the original system, can be used to reduce the computational complexity. There are several difficulties in our problems. First, because of the use of fBm, the techniques of martingale problem formulation can no longer be used. Second, there is no strong solution available and the underlying FSPDEs admit only a unique mild solution. Moreover, although the regime-switching enlarges the applicability of the underlying systems, to treat such systems is more difficult. To overcome the difficulties, fixed point theorem together with the use of stopping time argument, and a semigroup approach are used.

1. Introduction

This work develops an averaging principle for functional stochastic partial differential equations (FSPDEs) driven by a fractional Brownian motion (fBm). In addition, our systems are modulated by random switching processes. The switching processes are used to model random environment and other random factors that are not presented in the continuous dynamics. The switching processes display both fast and slow motions resulting in a natural two-time scales. Taking advantages of the fast and slow motions, we focus on the limit behavior of the underlying process, in which the original complex system is replaced by a much simpler averaged system.

There is an extensive literature on averaging principles for stochastic differential equations (SDEs); see for examples, Freidlin and Wentzell [1], Khasminskii [2–5], Yin and Zhang [6], Xu [7–9]. Recently, averaging principles for SPDEs have also drawn much attention; see, for examples, Duan [10], Wang [11], Pei [12], Fu and co-workers [13–15]. Cerrai and Freidlin [16] used an approach which is based on Kolmogorov equations and martingale solutions of stochastic equations to

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http://dx.doi.org/10.1016/j.nahs.2017.08.008
1751-570X/

derive averaging results for two-time-scale stochastic reaction diffusion equations whose additive noise (Brownian noise) is included in the fast motion. Cerrai [17] extended the results in [16] to the general case of multiplicative Brownian noises case where the diffusion in the slow equation depends on both slow and fast variables. Then, Bréhier [18] investigated a system of stochastic evolution equations of parabolic type with two-time scales. Very recently, Xu et al. [19] established averaging principles for two time-scale jump–diffusion process in the sense of mean-square. Pei, Xu and Wu [12] considered the averaging principle for stochastic hyperbolic–parabolic equations driven by Poisson random measures with slow and fast time-scales.

During the past decades, a great deal of attention has been paid to the study of SPDEs because of their applicability to ecology, economics, physics, chemistry, biology, population genetics, and finance. For references on SPDEs, we mention the work [20–24] and references therein. Recently, hybrid systems including random effects that combine continuous states and discrete events have received much attention. For instance, we consider a one-dimensional rod whose ends are fixed at 0° and whose sides are insulated. Assume that there is an exothermic reaction taking place inside the rod with heat being produced proportionally to the temperature. The temperature u in the rod can be modeled by

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + cu, t > 0, x \in (0, \pi), \\ u(t, 0) = u(t, \pi) = 0, u(t, x) = u_0(x), \end{cases} \tag{1.1}$$

where $u = u(t, x)$ and c is a constant dependent on the rate of reaction. System (1.1) switches from one mode to another in a random way when the abrupt changes are experienced in the structure and parameters caused by phenomena such as component failures or repairs, communication link interruption, or abrupt environmental disturbances. A hybrid system driven by a continuous-time Markov chain can be applied to such a situation. The system under regime switching could be described by the following random model

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} + c(r(t))u, t > 0, x \in (0, \pi), \\ u(t, 0) = u(t, \pi) = 0, u(t, x) = u_0(x), r(0) = r_0, \end{cases} \tag{1.2}$$

where $r(t)$ is a continuous-time Markov chain with a finite state space $\mathbb{S}$ and $c(\cdot) : \mathbb{S} \to \mathbb{R}$. For further details, we refer Bao et al. [25]. Taking into consideration of random environment and other random factors, it is widely known that such regime-switching formulation (1.2) is an effective way of modeling many practical situations.

Concerning the development of the singularly perturbed SPDEs, the noise processes considered to date are mainly martingale processes. It is well known that both Wiener processes and centered Poisson processes are martingale processes, whereas a fBm is not a semi-martingale. Fractional Brownian motions appear naturally and play an important role in the modeling of many complex phenomena in applications when the systems are subject to "rough" external force. Distinct significantly from the standard Brownian motion, a fBm is not a semi-martingale and it also exhibits power scaling with Hurst exponent $H \in (0, 1)$ [26]. When $H = \frac{1}{2}$ the fBm becomes the standard Brownian motion. This process was introduced by Kolmogorov in [27] and later studied by Mandelbrot and Van Ness in [28]. Due to the fact that the fBm is self-similar and has long-range dependence, this process is useful to model driving noise arising in various fields, especially mathematical finance, hydrology and queueing theory [29–31]. From the classical theory point of view, the difficulties with fBm are due to the lack of the martingale property. As a result, the properties of usual stochastic integral do not apply and we lost the tools of using such inequalities as Burkholder–Davis–Gundy inequality which is crucial for stochastic systems driven by Brownian motions. Such processes have received increasing attention recently. For example, Kou and Xie [32] considered a theoretical model for subdiffusion based on the generalized Langevin equation with fractional Gaussian noise which can be identified with the derivative of fBm, which provides an appropriate physical description for equilibrium fluctuation with a long memory in a closed system. Zhang, Bai, and Zhou [33] considered a classical risk model that is perturbed by a standard fBm with Hurst parameter $H \in (\frac{1}{2}, 1)$. Garrido-Atienza et al. [34] studied unstable invariant manifolds for stochastic PDEs driven by a fBm. Tindel [35] investigated the existence and uniqueness of the solutions to linear stochastic evolution equations driven by an infinite-dimensional fBm. Maslowski and Nualart [36] derived the nonlinear stochastic evolution equations in a Hilbert space driven by a cylindrical fBm with Hurst parameter $H > \frac{1}{2}$ and nuclear covariance operator. Nualart and Ouknine [37] showed the existence and uniqueness of a solution for a quasilinear parabolic equation in one dimension driven by an additive fractional white noise. Caraballo et al. [38] studied the existence and exponential behavior of solutions to stochastic delay evolution equations with a fBm. Fan and Yuan [39] calculated the Lyapunov exponents of PDEs driven by fractional noise with Markovian switching. In particular, Xu [40,41] investigated averaging principles for stochastic dynamical systems with or without delay driven by fBm. Xu, Pei and Guo [42] studied the stochastic averaging of slow–fast dynamical systems driven by fBm with the Hurst parameter H in the interval $(\frac{1}{2}, 1)$. Nevertheless, FSPDEs driven by a fBm with two-time scale Markovian switching have not been considered yet to date to the best of our knowledge. With the motivation of the previous references, we develop averaging principles for a class of FSPDEs driven by fBms of the form:

$$\begin{cases} dX(t) = \{AX(t) + b(t, X_t, r(t))\}dt + g(t)dB_Q^H(t), t \in [0, T], \\ X(t) = \psi(t), t \in [-\zeta, 0], \end{cases} \tag{1.3}$$

where X_t is known as the segment process with $X_t \in C([-\zeta, 0]; L^q(\Omega; V))$, and the initial data $\psi(t) \in C([-\zeta, 0]; L^q(\Omega; V))$, $t \in [-\zeta, 0]$ and $r(t)$ is a continuous-time Markov chain with a finite state space $\mathbb{S}$ which is independent of the fBm $B_Q^H(t)$.