Nonlinear Analysis: Hybrid Systems 27 (2018) 157–173

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

The existence and stability analyses of periodic orbits in 3-dimensional piecewise affine systems

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ARTICLE INFO

Article history:
Received 7 December 2016
Accepted 21 August 2017

Keywords:
Periodic orbits
Sink
Saddle
Poincaré map
Fixed points
Piecewise affine systems

ABSTRACT

This paper investigates the existence and stability of periodic orbits in a class of 3-dimensional piecewise affine systems with two zones. Two results on the existence of periodic orbits are obtained by applying some fixed points theorems to a Poincaré map. Precisely, we prove the existence of a periodic sink in one case and a periodic saddle in another case. Finally, some concrete examples are given to illustrate the main results.

1. Introduction

Due to the important applications in circuit design, control theory, computer science and biomolecular networks [1–5], hybrid systems have recently been one of the centers of active research. Piecewise smooth (PS) systems (flows), such as switched systems [6], can be viewed as a special case of hybrid dynamical systems and have been used to model a variety of practical systems, such as design of chaos generator, sliding mode control, robotic control systems, cell cycle regulation and so on [1,7–10]. Besides their enormous practical importance, PS systems also provoke a fascinating and highly challenging area of the study of a general theory of PS systems.

Periodic phenomena are quite pervasive in PS systems. Therefore the study of existence and stability of periodic cycles in the PS systems is of much interest both from theoretical and practical perspectives [11–22]. However, as is well known, searching for periodic trajectories is not easy even for only a planar PS system. In recent years, many classical methods are used to handle phenomena such as Hopf bifurcation, Homoclinic bifurcation and so on in smooth systems and are developed to study PS systems, especially planar PS systems. For instance, the Hopf bifurcation for planar PS systems was studied in [16–18], and some sufficient conditions for the appearance of limit cycles are obtained; the Melnikov method are extended to piecewise near-Hamiltonian in [19–22], and some significant results on Hopf bifurcation or homoclinic bifurcation leading to limit cycles are given by constructing the first Melnikov function. From the literatures mentioned above, we know that most of the previous work on periodic orbits focuses on planar PS systems. The topic of periodic orbits in higher dimensional PS systems, for instance, 3-dimensional PS flows, is touched seldom. Especially, few techniques are developed to deal with the existence of saddle periodic trajectories in higher dimensional PS systems. Thus, it is necessary to pay more attention to the periodic motion in higher dimensional PS systems.

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http://dx.doi.org/10.1016/j.nahs.2017.08.010
1751-570X/

In this paper, we will investigate the existence and stability of periodic orbits in a class of 3-dimensional piecewise affine systems possessing a switching plane and two admissible equilibrium points with purely real eigenvalues. By studying the existence of the fixed points of an appropriate Poincaré map defined in a small subset of the switching plane, two results on the existence of the periodic orbits are obtained by using the Brouwer fixed points theorem and a "saddle type" fixed points theorem, respectively. Furthermore, the stability analysis are carried out by analyzing detailedly the eigenvalues of the Poincaré map at the fixed points, showing that in one case the periodic orbit is a periodic sink, in other one case the periodic orbit is a periodic saddle. Finally, some examples are presented to show our main results.

The rest of this paper is organized as follows. In Section 2, we give the considered systems and two main results of this paper. In Section 3, we give some preliminaries as a preparation for the proofs of the main results. In Sections 4 and 5, we show detailedly the proofs of the main results in Section 2, respectively. In Section 6, we give some examples to illustrate the main results. In Section 7, we give the conclusions.

2. Main results

Consider the following 3-dimensional piecewise affine systems with a parameter

$$\dot{\mathbf{x}} = \begin{cases} A(\mathbf{x}-\mathbf{p}), & \text{if } \mathbf{c}_\mu^T\mathbf{x} \le d \\ B(\mathbf{x}-\mathbf{q}), & \text{if } \mathbf{c}_\mu^T\mathbf{x} > d \end{cases}, \tag{1}$$

where $\mathbf{x} = (x,\ y,\ z)^T \in \mathbb{R}^3$ is a vector of state variables, $\mathbf{p} = (p_x, p_y, p_z)^T$ and $\mathbf{q} = (q_x, q_y, q_z)^T$ are both constant vectors in $\mathbb{R}^3$, d is a positive constant, and $\mathbf{c}_\mu = (1-\mu, 0, 1)^T$ with μ being a small parameter. Suppose that

$$A = P\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}P^{-1},\quad B = Q\begin{pmatrix} \gamma_1 & 0 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix}Q^{-1}, \tag{2}$$

where $\lambda_2 < \lambda_1 < 0, \lambda_3 > 0, \gamma_2 < \gamma_1 < 0$ and $\gamma_3 > 0$, furthermore, P and Q are both invertible matrixes given by

$$P = (\xi_1, \xi_2, \xi_3), Q = (\zeta_1, \zeta_2, \zeta_3), \tag{3}$$

respectively. Obviously, $\xi_{1,2,3}$ are three linear independent vectors in $\mathbb{R}^3$, the same to $\zeta_{1,2,3}$. Without loss of generality, we suppose both of the z-coordinates of ξ_3 and ζ_3 are positive. Let

$$\Sigma_\mu = \{\mathbf{x} = (x, y, z)^T \in \mathbb{R}^3 | \mathbf{c}_\mu^T\mathbf{x} = d\}, \tag{4}$$

$$\Sigma_\mu^- = \{\mathbf{x} = (x, y, z)^T \in \mathbb{R}^3 | \mathbf{c}_\mu^T\mathbf{x} < d\}, \tag{5}$$

$$\Sigma_\mu^+ = \{\mathbf{x} = (x, y, z)^T \in \mathbb{R}^3 | \mathbf{c}_\mu^T\mathbf{x} > d\}. \tag{6}$$

It should be emphasized here if the sliding motion occurs in the switching surface Σ_μ, then the sliding solution is defined according to Filippov's convex method [1,23].

Now, we give the main results of this paper by the following two theorems, which will be proved in Sections 4, 5 respectively.

Theorem 1. *For system (1), suppose that all of the following conditions are satisfied:*

(i) $\mathbf{p}$ *and* $\mathbf{q}$ *are both the admissible equilibrium points of system (1) with* $\mu = 0$, *i.e.,*

$$\mathbf{c}_0^T\mathbf{p} < d, \mathbf{c}_0^T\mathbf{q} > d.$$

(ii) *there exists a constant* $l_1 > 0$ *such that*

$$\mathbf{p}_0 = \mathbf{p} + l_1\xi_3 \in \{(0, 0, d)^T + k(0, 1, 0)^T | k \in \mathbb{R}\} \cap \{\mathbf{q} + k_1\zeta_1 + k_2\zeta_2 | k_{1,2} \in \mathbb{R}\}$$

(iii) *there exists a constant* $l_2 < 0$ *such that*

$$\mathbf{q}_0 = \mathbf{q} + l_2\zeta_3 \in \Sigma_0 \cap \{\mathbf{p} + k_1\xi_1 + k_2\xi_2 | k_{1,2} \in \mathbb{R}\}.$$

(iv) $\mathbf{c}_0^T A(\mathbf{q}_0 - \mathbf{p}) < 0, \mathbf{c}_0^T B(\mathbf{p}_0 - \mathbf{q}) > 0.$

Then,

(a) *if* $\frac{|\lambda_1||\gamma_1|}{\lambda_3\gamma_3} > 1$, *there exists at least a periodic orbit for any small* $\mu > 0$.
(b) *if* $\frac{|\lambda_1||\gamma_2|}{\lambda_3\gamma_3} + \frac{|\lambda_1|}{\lambda_3} < 1$, *there exists at least a periodic orbit for any small* $\mu < 0$.