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Persistence and extinction of a modified Leslie–Gower Holling-type II stochastic predator–prey model with impulsive toxicant input in polluted environments

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ABSTRACT

In this paper, a stochastic predator–prey model with modified Leslie–Gower Holling-type II schemes and impulsive toxicant input in polluted environments is developed and analyzed. The threshold between persistence in the mean and extinction is established for each population. Some simulation figures are also introduced to illustrate the theoretical results.

1. Introduction

Environmental pollution caused by agriculture, industries and other human activities has become an issue of serious international concern. Contaminants in the environment have threatened humans and other living organisms. For example, Amoco Cadiz oil spill incident killed 260,000 tonnes of marine animals in the first month [1]. Bhopal chemical accident caused more than 10,000 people die in the first few days [2]. In the United States, more than 72 million birds are killed by pesticides each year, estimated by the US Fish and Wildlife Service [3].

These accidents motivate the scholars to investigate the effect of contaminants on the survival of the species by using mathematical modeling. Hallam and his co-workers did pioneering works in [4–6]; they developed and analyzed several deterministic single-species models in polluted environments. From then on, many deterministic population models in polluted environments were developed and studied, see e.g. [7–15]. Particularly, taking into account the fact that contaminants are often emitted in regular pulses (for example, pesticides and heavy metals [16]), Yang et al. [13] proposed a deterministic two-species Lotka–Volterra predator–prey model with pulse toxicant input in polluted environments, and established the persistence-and-extinction threshold for each species.

The model in [13] is based on the traditional Lotka–Volterra predator–prey model. Several scholars [17,18] have pointed out that the traditional Lotka–Volterra predator–prey model has several limitations, and more realistic models should take the functional response into account. Thus in recent years, both applied mathematicians and ecologists have paid attention to predator–prey models with many kinds of functional response. Particularly, predator–prey model with modified Leslie–Gower and Holling-type II schemes [19], which can describe the fact that the reduction of the predator population may have a reciprocal relationship with per capita availability of its prey, has received great attention and has been studied

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extensively (see e.g., [20–25]). However, so far as our knowledge is concerned, no results related to predator–prey models with modified Leslie–Gower and Holling-type II schemes in polluted environments have been reported. On the other hand, the growth of population in the natural world is inevitably affected by random fluctuations [26]. And several authors have pointed out that the random fluctuations may change the dynamics of population models greatly. For example, Mao, Marion and Renshaw [27] have shown that the random fluctuations can suppress a potential population explosion. Motivated by these, in this paper we consider a modified Leslie–Gower Holling-type II stochastic predator–prey model with impulsive toxicant input in polluted environments.

This paper is organized as follows. In Section 2, we propose our stochastic model. Then in Section 3, we establish the threshold between persistence in the mean and extinction for each species. We introduce several numerical simulations to illustrate the main results in Section 4. The paper ends with some interesting conclusions in Section 5.

2. The model

Holling type II functional response, which measures the average feeding rate of a predator when it spends some time finding and processing the prey, has served as the basis for lots of literature on predator–prey theory [28]. The classical predator–prey model with Holling type II functional response can be expressed as follows:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\Big(r_1 - \eta x(t)\Big) - \frac{\kappa x(t)y(t)}{1+\lambda x(t)}, \quad \frac{\mathrm{d}y(t)}{\mathrm{d}t} = -\varphi y(t) + \frac{\mu\kappa x(t)y(t)}{1+\lambda x(t)},$$

where $x(t)$ and $y(t)$ stand for the sizes of the prey population and the predator population respectively; r_1, a, κ, λ, φ and μ are positive constants. r_1 stands for the growth rate of the prey; η is the competitive strength among individuals of the prey; κ and λ measure the effects of capture rate and handling time (the time processing the prey [28]), respectively; φ represents the death rate of the predator; μ is the food conversion rate of the predator.

Leslie [29] pointed out that the reduction of a predator population is reciprocal to the per capita availability of its preferred prey, and proposed a predator–prey model in which the environmental carrying capacity of the predator is proportional to the number of the prey [30], that is,

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = ry(t)\left(1 - \frac{y(t)}{\alpha x(t)}\right),$$

where $r > 0$ is the growth rate of the predator and $\alpha > 0$ stands for the conversion factor of prey into predator. $y/\alpha x$ is called the Leslie–Gower term [30,31], which measures the loss in the predator population owing to the rarity of its preferred prey. On the other hand, when the preferred prey x is severely rare, the predator y can capture other populations but its growth rate will be limited because its preferred prey (x) is severely rare [19]. Taking this fact into account, Aziz-Alaoui and Okiye [19] added a positive constant to the denominator of the Leslie–Gower term, and proposed the following predator–prey model with *modified* Leslie–Gower Holling-type II schemes:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\left(r_1 - \eta x(t) - \frac{cy(t)}{\theta + x(t)}\right), \quad \frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)\left(r_2 - \frac{fy(t)}{\theta + x(t)}\right), \tag{1}$$

where $c,\ \theta,\ r_2$ and f are positive constants. c represents the per capita reduction rate of the prey due to the capture of the predator. θ is the capacity of food provision of the environment except x. r_2 denotes the growth rate of the predator. f is the per capita reduction rate of the predator.

Suppose that the living organisms absorb environmental toxicants into their bodies [4–6]. Let $T_{10}(t)$ and $T_{20}(t)$ be the concentration of toxicant in the prey organism and predator organism at time t, respectively. Suppose that the growth rate, r_i, is an affine function of $T_{i0}(t)$ [4,7,8]:

$$r_i \to r_{i0} - r_{i1}T_{i0}(t),\ i = 1, 2.$$

Then model (1) becomes

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)\left(r_{10} - r_{11}T_{10}(t) - \eta x(t) - \dfrac{cy(t)}{\theta + x(t)}\right), \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)\left(r_{20} - r_{21}T_{20}(t) - \dfrac{fy(t)}{\theta + x(t)}\right). \end{cases}$$

Now let us take the white noise perturbations of the growth rates into account. In fact, the growth rates of the species are often affected by the random fluctuations [26]. Generally speaking, the random fluctuations in the environment can be