Nonlinear Analysis: Hybrid Systems 27 (2018) 258–270

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Dynamic behavior analysis of SIVS epidemic models with state-dependent pulse vaccination ☆

Lin-Fei Nie *, Jing-Yun Shen, Chen-Xia Yang

College of Mathematics and Systems Science, Xinjiang University, Urumqi 830046, PR China

HIGHLIGHTS

- Two novel epidemical models with state feedback control strategies are introduced.
- Some sufficient conditions on the existence and orbit stability of positive periodic solution are presented.
- The existence and orbit stability of semi-trivial periodic solution are proved, which show that the disease is dying out.
- Numerical simulations are carried out to illustrate the main results.

ARTICLE INFO

Article history:
Received 12 August 2015
Accepted 7 August 2017

Keywords:
SIVS epidemic model
State-dependent pulse
Order-*k* periodic solution
Semi-trivial periodic solution
Orbital stability

ABSTRACT

To prevent and control the spread of infectious disease, in this paper, we formulate two susceptible–infected–vaccinated–susceptible (SIVS) epidemic models, where state-dependent pulse vaccination control strategies are introduced. The first model incorporates the density of infected individuals as the monitoring threshold value and some sufficient conditions for the existence and orbital stability of order-1 or order-2 periodic solutions are obtained by using the Poincaré map and qualitative theory of ordinary differential equations. Further, using the density of susceptible individuals as the monitoring threshold value, the second model with state-dependent pulse vaccination is proposed, and the existence and stability of the semi-trivial periodic solution and order-1 periodic solution can be obtained via utilizing the analog of the Poincaré criterion. Theoretical results imply that a state-dependent pulse vaccination strategy can eliminate the spread of infectious disease or keep the density of infected individuals at a desired low level for a long time. Finally, numerical simulations are given to verify the correctness of the theoretical results and to obtain the highest efficiency of state-dependent pulse control strategy.
© 2017 Elsevier Ltd. All rights reserved.

1. Introduction

Infectious diseases are caused by microorganisms such as viruses, bacteria, fungi, or parasites and can spread between individuals. These diseases can be spread, directly or indirectly, from one person to another. Some pathogens are spread through the air or through surface contact. Every year, millions of humans suffer or die from various infectious diseases. The control, and hence, eradication of infectious disease has been the subject of scientific work in various fields, including medicine and mathematics. In particular, mathematicians have proposed various dynamic models to study the transmission

☆ This research is partially supported by the National Natural Science Foundation of China (Grant No. 11461067), the Natural Science Foundation of Xinjiang (Grant Nos. 2016D01C046 and 2016D03022).
* Corresponding author.
E-mail address: lfnie@163.com (L.-F. Nie).

https://doi.org/10.1016/j.nahs.2017.08.004
1751-570X/© 2017 Elsevier Ltd. All rights reserved.

mechanism of infectious diseases to predict the future course of an outbreak and to evaluate strategies to control the spread of infectious diseases. Among them, mathematical models of differential equations have attracted the attention of many scholars, and the main study contents include the non-negativeness and boundedness of solution, the persistence and extinction of infectious diseases as well as the threshold condition of the basic reproductive number, and so on. We refer to [1–7] and the references therein.

It is well known that vaccination is regarded as the most effective measure to prevent and control the spread of infectious diseases, such as rabies, yellow fever, poliomyelitis, measles [8], and so on. Considering that vaccinations take place only at certain times, Shulgin et al. [9] put forth the theory of fixed-time pulse vaccination strategy (FPVS), which consists of periodic repetitions of pulse vaccination in a population, and showed that the infection-free solution can exist and be stable. Soon after, theories and applications of pulse vaccination with fixed time have been a hot topic of epidemic dynamic models. For example, d'Onofrio [10] proposed a susceptible–exposed–infected–removed (SEIR) epidemic model with FPVS and obtained the local asymptotic stability and global asymptotic stability of the disease-free periodic solution. Liu et al. [11] established two susceptible–vaccinated–infected–removed (SVIR) models to describe continuous vaccination strategy and FPVS, respectively. More various interesting results can be found in [12–15] and the references therein. Theoretical results show that FPVS can be distinguished from the conventional strategies in leading to disease eradication at relatively low values of vaccination.

Recently, however, increasing amounts of data have shown that the eradication of some infectious diseases (such as cholera, influenza, pulmonary tuberculosis, bacillary dysentery, brucellosis, and so on) in a short time is sometimes difficult both practically and economically. From an economic standpoint, control measures may not be necessary if the density of infected individuals in the population is relatively small, or the disease does not cause serious harm to the society. Therefore, FPVS is not the most effective or economical way to eliminate these infectious diseases because the vaccination strategies are taken at some fixed times and are not dependent on the state of infectious disease. In general, to prevent the spread of infectious disease, the public may be encouraged to get vaccinated when the number of infected individuals is relatively high (for example, reaching a critical level). This type of control strategy depends on the state of infected individuals (or susceptible individuals) and is called a state-dependent pulse vaccination strategy. Based on this idea, Tang et al. [16] proposed a general epidemic model incorporating state-dependent pulse control using a combination of pulse vaccination of susceptible individuals and treatment (or isolation or both) of the infected individuals and discussed the effects of implementing state-dependent pulse vaccination and/or treatment (or isolation) to evaluate the feasibility and expense of disease control over an extended period. Nie et al. [17–19] applied state-dependent pulse control strategies to a susceptible–infected–removed (SIR) model, susceptible–infected–removed–susceptible (SIRS) model, and a viral model, and obtained some sufficient conditions for the existence and stability of positive periodic solutions by the method of qualitative analysis, Poincaré map, Poincaré criterion, and other analysis methods. In addition, a state-dependent pulse control strategy is also commonly used in mathematical biology owing to its low-cost, high-efficiency, and feasible nature, for example, in population dynamic models [20,21], chemostat dynamic models [22,23], microbial growth models [24,25], cylindrical dynamic systems [26], and so on.

Motivated by the above discussion, the main purpose of this paper is to analyze whether state-dependent pulse vaccination can be used to effectively prevent and control the spread of infectious disease and to address how these key factors (including the strength of vaccination and threshold value) affect the control of infectious disease. The rest of this paper is organized as follows. In Section 2, two susceptible–infected–vaccinated–susceptible (SIVS) models with state-dependent pulse vaccination are proposed, and some preliminary definitions and lemmas are also introduced. In Section 3, the existence and orbital stability of an order-1 or order-2 periodic solution for the first model is presented, where the density of infected individuals is used as the control threshold value. The existence and stability of the semi-trivial periodic solution and order-1 periodic solution of the other model are reported in Section 4, where control strategies are taken when the density of susceptible individuals reaches a risk threshold value. In Section 5, by carrying out some numerical simulations, we try to confirm the main theoretical results to illustrate some key factors to prevent and control the spread of infectious disease. Some concluding remarks are presented in the last section.

2. Model formulation and preliminaries

In this section, we propose two epidemic models with state-dependent pulse vaccination. For convenience, we separate the human population into three classes, susceptible, infected, and vaccinated, with sizes denoted by $S(t)$, $I(t)$, and $V(t)$, respectively. The following model is a classic SIVS model with fixed-time pulse vaccination, which was developed in [27] as follows:

$$\begin{cases} \left.\begin{aligned} \frac{\mathrm{d}S(t)}{\mathrm{d}t} &= \mu N(t) - \beta S(t)I(t) - \mu S(t) + cI(t) + \theta V(t) \\ \frac{\mathrm{d}I(t)}{\mathrm{d}t} &= \beta(S(t) + \sigma V(t))I(t) - (\mu + c)I(t) \\ \frac{\mathrm{d}V(t)}{\mathrm{d}t} &= -\sigma\beta V(t)I(t) - (\mu + \theta)V(t) \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} S(nT^+) &= (1-\omega)S(nT) \\ I(nT^+) &= I(nT) \\ V(nT^+) &= V(nT) + \omega S(nT) \end{aligned}\right\} t = nT,\ n = 0, 1, 2, \ldots, \end{cases} \tag{1}$$

Download English Version:

https://daneshyari.com/en/article/5471989

Download Persian Version:

https://daneshyari.com/article/5471989

Daneshyari.com