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Computational method for optimal control of switched systems with input and state constraints

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HIGHLIGHTS

- A gradient algorithm is proposed for solving the switching time optimization problem.
- A filled function is developed for solving the mode sequence optimization problem.
- Convergence results indicate that the proposed algorithm is globally convergent.
- Our algorithm converges faster and yields a better value than existing methods.

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ABSTRACT

In this paper, we consider an optimal control problem of switched systems with input and state constraints. Since the complexity of such constraint and switching laws, it is difficult to solve the problem using standard optimization techniques. In addition, although conjugate gradient algorithms are very useful for solving nonlinear optimization problem, in practical implementations, the existing Wolfe condition may never be satisfied due to the existence of numerical errors. And the mode insertion technique only leads to suboptimal solutions, due to only certain mode insertions being considered. Thus, based on an improved conjugate gradient algorithm and a discrete filled function method, an improved bi-level algorithm is proposed to solve this optimization problem. Convergence results indicate that the proposed algorithm is globally convergent. Three numerical examples are solved to illustrate the proposed algorithm converges faster and yields a better cost function value than existing bi-level algorithms.

1. Introduction

Switched systems are a particular class of hybrid systems which consist of several modes and a switching law specifying the active mode at each time instant. Many complex processes can be modeled as such systems. For example, automobiles employing different gears [1], biological systems [2], manufacturing systems [3], and conditions where a control module has to switch its attention among many subsystems [4,5].

Recently, switched system optimal control problems which involve finding a switching sequence and an continuous input such that a cost function is optimized, have attracted researchers from various fields in science and engineering due

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to such problems' significance in theory and application. Some contributions have been devoted to the existence of optimal solutions and development of methods for finding an optimal solution. With a given mode sequence, an optimal control problem for switched systems is discussed in [6]. The idea is that the problem is transcribed into an equivalent optimization problem based on parameterization of the switching instants and then obtain the derivatives of the cost function with respect to the switching times based on the solution of a two point boundary value differential algebraic equation. Later, the same authors investigate a time optimal control problem for integrator switched systems with polyhedral state constraint subsets based on the graph representation [7]. Boccadoro et al. [8] consider an optimal switching surfaces problem, and derive the gradient of the cost function. Then a gradient-descent algorithm is used for solving an obstacle-avoidance problem in robotics. By embedding approach, the optimal control problem of two-switched nonlinear systems which imposes no restriction on the number of switching or the mode sequence is studied [9]. In [10], a novel numerical algorithm based on differential transformation is proposed for solving the optimal control problem of a class of switched systems with a predefined mode sequence. The work in [11] consider the optimal control problem of autonomous switched nonlinear systems and develop a bi-level hierarchical algorithm: at the higher level, the algorithm updates the mode sequence by employing a single mode insertion technique, and at the lower level, the algorithm assumes a fixed mode sequence and minimizes the cost functional over the switching times. An approach for computing the derivative of the optimal value function based on analyzing differentiability of the cost function is presented in [12]. The recent work [13–19] present some algorithms for solving several classes of discrete-time switched systems optimal control problems. For more discussions on various literature results, the reader may refer to [20–34], and the references therein.

Many dynamic control processes have requirements that must be satisfied at every point along a trajectory, such as container cranes [35], aircraft trajectory planning [36], and sensor scheduling [37]. In this paper, we consider an optimal control problem of switched systems with input and state constraints. Note that one such constraint (continuous-time constraint) is equivalent to an uncountable number of conventional constraints, it is difficult to numerically integrate the switched system with the initial condition since the switching times are unknown, and the optimal control problem of switched systems is essentially a mixed integer programming problem. Thus, it is difficult to solve the problem by conventional optimization techniques. To overcome the difficulty, we adopt a bi-level algorithm to divide the problem into two nonlinear optimization problems: one is a continuous time optimal control problem and the other is a discrete optimization problem. For the first question, with a given mode sequence, our main task is finding the optimal switching instants. To solve the problem, we transform the inequality constraint into an equality constraint which is smoothed using a twice continuously differentiable function and treated as a penalty function. Furthermore, the penalty function is appended to the cost functional to form an augmented cost function, giving rise to an approximate optimization problem that can be solved by any gradient-based method, such as Newton method, Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm et al. Note that the iteration of the Newton method which need to compute Hessian matrix is time-consuming, while the BFGS algorithm may not converge. Generally speaking, conjugate gradient algorithms are very useful for solving such problem, which is a nonlinear optimization problem, especially when the dimension of the problem is large. However, existing conjugate gradient algorithms may generate many short steps without making productive progress to the minimum, e.g., Hestenes–Stiefel method [38]. And when applying the Hestenes–Stiefel method to solve the problem with quadratic objective functions, the equality $g_k^{\mathrm{T}} g_{k+1} = 0$ holds, while, for general nonlinear objective functions, the value $g_k^{\mathrm{T}} g_{k+1}$ may not be zero. In addition, a step-size α_k needs to be determined along the direction d_k at each iteration of the Hestenes–Stiefel method. For example, it is often required to satisfy the Wolfe conditions or some of its variants such as the strong Wolfe conditions. Theoretically, there must exist some positive α_k satisfying the Wolfe conditions and its variants if d_k is a descent direction. However, in practical implementations, the Wolfe condition may never be satisfied due to the existence of numerical errors. Thus, to solve this problem, by a novel restart strategy and a generalized improved Wolfe line search condition, we introduce an improved conjugate gradient algorithm. As to the second problem, the mode insertion technique [33] is proposed to solve it. However, the algorithm only leads to suboptimal solutions, due to only certain mode insertions being considered. To get global optimal solutions, we develop a discrete filled function method. Our main contribution is, by a novel restart strategy, a generalized improved Wolfe line search condition and an improved discrete filled function, an improved bi-level algorithm is developed for solving the problem. Convergence results indicate that the proposed algorithm is globally convergent. Finally, three numerical examples demonstrate that our method converges faster and yields a better cost function value than existing bi-level algorithms.

The rest of this article is organized as follows. In Section 2, we formulate the optimal control problem of switched systems with input and state constraints studied in the paper. Optimization algorithms are proposed in Sections 3 and 4. The effectiveness of our algorithm is shown in Section 5.

2. Problem formulation

Let $T > 0$ be a fixed terminal time and define

$$\mathcal{F} = \left\{ \sigma \in R^r : \ a_i \leq \sigma_i \leq b_i, \ i = 1, \ 2, \ \cdots, \ r \right\},$$

where, for each $i = 1, \ 2, \ \cdots, \ r$, a_i and b_i $(a_i < b_i)$ are given real constants.