Nonlinear Analysis: Hybrid Systems 26 (2017) 19–37

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Dynamics analysis and numerical simulations of a stochastic non-autonomous predator–prey system with impulsive effects

Shengqiang Zhang [a], Xinzhu Meng [a,b,c,*], Tao Feng [a,c], Tonghua Zhang [d]

[a] College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, PR China
[b] State Key Laboratory of Mining Disaster Prevention and Control Co-founded by Shandong Province and the Ministry of Science and Technology, Shandong University of Science and Technology, Qingdao 266590, PR China
[c] Key Laboratory of Jiangxi Province for Numerical Simulation and Emulation Techniques, Gannan Normal University, Ganzhou 341000, PR China
[d] Department of Mathematics, Swinburne University of Technology, Hawthorn, VIC 3122, Australia

HIGHLIGHTS

- An impulsive stochastic non-autonomous Lotka–Volterra predator–prey model is proposed and investigated.
- The existence and global attraction of the positive periodic solution of the subsystem are proved.
- The thresholds for stochastic persistence and extinction of the system are obtained.
- The developed numerical method for simulating the global stochastic dynamics of the model.

ARTICLE INFO

Article history:
Received 8 January 2017
Accepted 2 April 2017

Keywords:
Non-autonomous predator–prey model
Impulsive effect
Stochastic persistence
Extinction
Stochastic perturbation

ABSTRACT

In this paper, we propose a stochastic non-autonomous Lotka–Volterra predator–prey model with impulsive effects and investigate its stochastic dynamics. We first prove that the subsystem of the system has a unique periodic solution which is globally attractive. Furthermore, we obtain the threshold value in the mean which governs the stochastic persistence and the extinction of the prey–predator system. Our results show that the stochastic noises and impulsive perturbations have crucial effects on the persistence and extinction of each species. Finally, we use the different stochastic noises and impulsive effects parameters to provide a series of numerical simulations to illustrate the analytical results.

1. Introduction

In the natural world, since populations are inevitably infected by various environmental noises such as white noise which is an important component in an ecosystem, and some authors (see e.g. [1–5]) have investigated the stochastic non-autonomous Lotka–Volterra predator–prey model with white noises:

$$\begin{cases} \mathrm{d}x_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)]\mathrm{d}t + \sigma_1(t)x_1(t)\mathrm{d}B_1(t), \\ \mathrm{d}x_2(t) = x_2(t)[r_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)]\mathrm{d}t + \sigma_2(t)x_2(t)\mathrm{d}B_2(t), \end{cases} \tag{1}$$

* Corresponding author.
E-mail address: mxz721106@sdust.edu.cn (X. Meng).

http://dx.doi.org/10.1016/j.nahs.2017.04.003
1751-570X/

where $x_1(t)$ and $x_2(t)$ represent the prey and predator populations at time t, respectively; $r_i(t)$ represents the intrinsic growth rate of the corresponding population at time t; $a_{11}(t)$ and $a_{22}(t)$ denote the density-dependent coefficients of the prey and predator populations, respectively; $a_{12}(t)$ stands for the capturing rate of the predator population and $a_{21}(t)$ represents the rate of conversion of nutrients into the reproduction of the predator population; $\sigma_1(t)$ and $\sigma_2(t)$ denote the coefficients of the environmental stochastic effects on the prey and predator populations, respectively; the standard Brownian motions $B_1(t)$ and $B_2(t)$ are mutually independent; $r_i(t)$, $a_{ij}(t)$ and $\sigma_i(t)$ $(i, j = 1, 2)$ are all continuous bounded functions on $R_+ := [0, +\infty)$.

Recently, the system (1) was studied extensively and some good results were obtained. In the stochastic model (1), Liu et al. [1] analyzed sufficient conditions for extinction and persistence in the mean, and Zu et al. [2] obtained sufficient conditions for the existence of one positive periodic solution. Rudnicki et al. [3,4] obtained that the distributions of the solutions are absolutely continuous, and investigated that the densities can converge in L^1 to an invariant density or converge weakly to a singular measure in the autonomous case of stochastic model (1). Nieto et al. [6] proposed random fixed point theorems in partially ordered metric spaces. There are many other papers on stochastic models with white noise perturbations, the readers please refer to [7–16] and references therein.

On the other hand, owing to many natural or man-made factors, impulsive effects appear widely in many evolution processes which are characterized by the fact that at some fixed times they undergo some discrete changes of relatively short time interval, involving such as earthquake, drought, flooding, fire, crop-dusting, planting, hunting, harvesting, etc. From the viewpoint of mathematics, those sudden changes could not be described continually. Therefore, the system (1) cannot illustrate these phenomena. To describe these phenomena more accurately, the study of impulsive differential equations have received significant attention, see [17–19] and references therein.

Many deterministic population dynamical models with impulse have been proposed and studied. Some important and interesting results on dynamical behavior for such systems have been obtained, see [20–28] and the references therein. Recently, the stability of stochastic differential equation (SDE) with impulsive effects has been studied by Sakthivel and Luo [29], Li and Sun [30]. As a matter of fact, some populations fluctuate over time and often show seasonal patterns. It is important and interesting for us to predict and control the development of populations according to take account of periodic variation in the stochastic model (1) and investigate the existence of periodic solutions. To the best of our knowledge, there is little amount of work on the stochastic non-autonomous Lotka–Volterra predator–prey model with impulsive perturbations [10].

Motivated by the above discussions, we investigate the following stochastic Lotka–Volterra predator–prey system with impulsive perturbations:

$$\begin{cases} \left.\begin{aligned} dx_1(t) &= x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t)]dt + \sigma_1(t)x_1(t)dB_1(t), \\ dx_2(t) &= x_2(t)[r_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t)]dt + \sigma_2(t)x_2(t)dB_2(t), \end{aligned}\right\} & t \neq t_k = k\tau,\ k \in Z^+, \\ \left.\begin{aligned} x_1(t_k^+) - x_1(t_k) &= \delta_{1k}x_1(t_k), \\ x_2(t_k^+) - x_2(t_k) &= \delta_{2k}x_2(t_k), \end{aligned}\right\} & t = t_k = k\tau,\ k \in Z^+, \end{cases} \tag{2}$$

where Z^+ denotes the set of positive integers, $r_i(t)$, $a_{ij}(t)$ and $\sigma_i(t)$ $(i, j = 1, 2)$ are continuous positive periodic functions with a common period $\tau > 0$, $0 < t_1 < t_2 < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$. We impose the following restriction on system (2) which is a reasonable way for biological meanings:

$$1 + \delta_{ik} > 0, \quad i = 1, 2,\ k \in Z^+.$$

When $\delta_{ik} > 0$, the impulsive perturbations stand for planting of the species, but if $\delta_{ik} < 0$, they stand for harvesting.

The main aims of this paper are to investigate how the white noises and impulsive perturbations affect the stochastic persistence and extinction of the system. The rest of the paper is arranged as follows. In Section 2, we prove that the subsystem of the system (2)exists a unique τ-periodic solution which is globally attractive. Then we show that when the white noises are small enough, the prey and predator populations are stochastically persistent but when the white noises are large enough, the two populations become extinct. In Section 3, the conclusions are given and our main results of stochastic persistence and extinction are illustrated through some examples and figures.

2. Stochastic persistence and extinction

In the section, we give some notations which can be used for definitions and some lemmas of our main results.

Throughout this paper, unless otherwise specified, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ stand for a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all P-null sets). For convenience, we define $f^u = \sup_{t\in R_+} f(t)$, $f^l = \inf_{t\in R_+} f(t)$, $\langle f(t)\rangle = \frac{1}{t}\int_0^t f(t)ds$. Here $f(t)$ is a continuous τ-periodic function, and $b_i(t) = r_i(t) - 0.5\sigma_i^2(t)$, $p_i(t) = \frac{1}{t}[\sum_{0<t_k<t} \ln(1 + \delta_{ik}) + \int_0^t b_i(s)ds]$, $i = 1, 2$. Moreover, we assume that a product equals unity if the number of factors is zero.

Before we investigate the stochastic persistence and extinction of system (2), we firstly prove the existence of global positive solution for the following subsystem of systems (2).