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Widening of the basins of attraction of a multistable switching dynamical system with the location of symmetric equilibria

L.J. Ontañón-García [a,*], E. Campos-Cantón [b]

[a] Coordinación Académica Región Altiplano Oeste, Universidad Autónoma de San Luis Potosí, Kilometro 1 carretera a Santo Domingo, 78600, Salinas de Hidalgo, San Luis Potosí, Mexico

[b] División de Matemáticas Aplicadas, Instituto Potosino de Investigación Científica y Tecnológica A.C., Camino a la Presa San José 2055 col. Lomas 4a Sección, 78216, San Luis Potosí, SLP, Mexico

HIGHLIGHTS

- Multistable solutions appear from the displacement of the equilibria of PWLS.
- Single, to multistable solution result due to the distance between the equilibria.
- Increasing the distance results in larger basin of attraction in the multistable system.

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ABSTRACT

A switching dynamical system by means of piecewise linear systems in $\mathbf{R}^3$ that presents multistability is presented. The flow of the system displays multi-scroll attractors due to the unstable hyperbolic focus-saddle equilibria with stability index of type I, i.e., a negative real eigenvalue and a pair of complex conjugated eigenvalues with positive real part. This class of systems is constructed by a discrete control mode changing the equilibrium point regarding the location of their states. The scrolls appear when the stable and unstable eigenspaces of each adjacent equilibrium point generate the stretching and folding mechanisms needed in chaos, i.e., the unstable manifold in the first subsystem carries the trajectory towards the stable manifold of the immediate adjacent subsystem.

The resulting attractors are located around four focus saddle equilibria. If the equilibria are located symmetrically to one of the axes and the distance between each equilibria is properly adjusted to generate two double-scroll chaotic attractors, the system can present from bistable to multistable parallel solutions regarding the position of their initial states. In addition the resulting basin of attraction presents a significatively widening when the distance between the equilibria of the parallel attractors is displaced.

1. Introduction

Switching dynamical systems are characterized by the mutual existence of continuous and discrete dynamic behavior. An interesting example of these systems is represented by piecewise linear (PWL) systems, particularly dissipative systems

* Corresponding author.
E-mail addresses: luis.ontanon@uaslp.mx (L.J. Ontañón-García), eric.campos@ipicyt.edu.mx (E. Campos-Cantón).

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with unstable dynamics which have been described by the *unstable dissipative systems* (UDS) theory [1–5]. This theory makes a characterization of the linear systems onto two types depending on the spectrum of their linear operator. Considering switching systems with phase space in $\mathbf{R}^3$, the stability index of type I refers to one negative real eigenvalue and one pair of complex conjugated eigenvalues with positive real part. If the location of the equilibria is adjusted taking into consideration this type of index, chaotic attractors with multi-scroll behavior can be generated.

The generation of multi-scroll attractors has attracted attention due to its applications ranging from building transmitter–receiver electronic devices, design pseudo-random number generators for cryptography and even neural state-space models [6,7]. The theory involved has been widely studied over the last decades. For example, in the work reported by Suykens in [8], n-double scrolls in the Chua's system were presented. After that there have been different approaches to yield multi-scroll attractors. These approaches vary from modifying the nonlinear part in the Chua's system, to using nonsmooth nonlinear functions, such as hysteresis, saturation, threshold, step functions, fractional-order systems and chaos entanglement [8–14]. The main difference between the UDS of the type I technique of multi-scroll generation in contrast to some other techniques, is that for each equilibrium point introduced to the system a new scroll emerges, presenting systems with the same number of scrolls than equilibrium points.

A well known property of chaotic systems is the highly sensibility to initial conditions, that is, chaotic systems which are initialized with small differences in initial conditions will result in diverging trajectories. However, these trajectories are confined to the same attractor and in most cases the only one of the system. This fact has led the scientists to study and design systems that present more than one stable solutions resulting in two or more attractors given a fixed set of parameters but different initial states. These multiple possible behaviors are isolated from each other and the term to refer to this property of the systems is defined as "multistability" [15,16]. A common method to study this property is by means of the basins of attraction of the system, which basically correspond to the long-time response of the system due to the location of different initial conditions. Generally, this method requires exhaustive computation time depending on the system properties and both the large scale of initial conditions and the time that the system is iterated [17,18].

There have been several reported applications or natural occurrences about multistability. However, biology and electronics are two of the most recurrent areas. A few examples considering the former area are described in the multiple behavioral patterns in neural dynamics or the multistable coordination dynamics [19,20]; in the dynamics of some biological central pattern generators through neurons connected in rings [21]; in the integrate-and-fire model of the neurons affected by white noise [22]. Now, consider the case of the area of electronics, in the circuit implementation of the digital selection of an active set of neurons or in the use of nonlinear reactances or negative resistors [23,24]. Switching dynamical systems describe interesting and common phenomena such as switching electrical circuits and systems involving both digital and analog components or physical systems affected by impact, sliding or friction forces [25,26], just to mention a few.

Multistable system, can either be generated by different techniques, or controlled in order to avoid their behavior depending on the application in which they are involved [27]. This also applies to multi-scroll system of the UDS type I which can present multistability depending on the location and separation between their equilibria as it will be addressed in the article. However the stable set of initial conditions, may become a restriction in their applications. In this context, here a new way to widen the basin of attraction of a bistable switching dynamical system considering only equilibria of the UDS type I whose solution presents parallel attractors will be described. This type of systems presents two interesting relationships between the symmetric equilibria and the resulting basins of attraction related to each individual stable system, (i) if a specific size is considered the system will present bistable or multistable solutions; (ii) the size of their basins of attraction will also change, increasing or diminishing with regard to the location of the symmetric equilibria.

The article is organized as follows: In Section 2 the general theory that envelops the generation of UDS is presented; In Section 3 the multi-scroll attractors are introduced due to the variation of the distance between the commutation surfaces; Section 4 contains the generation of multistable systems with the widening of the basin of attraction. And finally conclusions are drawn in Section 5.

2. UDS theory

Following the same structure as in [3–5], consider the class of switching linear system given by

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}, \tag{1}$$

where $\mathbf{X} = [x_1, x_2, x_3]^T \in \mathbf{R}^3$ is the state vector, $\mathbf{B} = [B_a, B_b, B_c]^T \in \mathbf{R}^3$ stands for a discrete real affine vector, $\mathbf{A} = [a_{ij}] \in \mathbf{R}^{3\times 3}$ with $i, j = 1, 2, 3$ denotes a nonsingular linear matrix. The equilibria will be located at $X^* = -\mathbf{A}^{-1}\mathbf{B}$. As described in Definition 2.1 in [3], a system with stability index of the type I will be addressed as a system of the UDS type I. Besides, the following considerations have to be made in order to call Eq. (1) an UDS of type I that in addition generates an attractor $\mathfrak{A}$.

(a) The linear part of the system must satisfy the dissipative condition $\sum_{i=1}^{3} \lambda_i < 0$, where λ_i, $i = 1, 2, 3$, are eigenvalues of $\mathbf{A}$. Consider also that one λ_i is a negative real eigenvalue, and two λ_i are complex conjugate eigenvalues with positive real part $Re\{\lambda_i\} > 0$, resulting in an unstable focus-saddle equilibrium X^*. This type of equilibria presents a stable manifold $M^s = span\{V_{\lambda_1}\} \in \mathbf{R}^3$ with a fast eigendirection and an unstable manifold $M^u = span\{V_{\lambda_2}, V_{\lambda_3}\} \in \mathbf{R}^3$ with a slow spiral eigendirection, where V_{λ_i} corresponds to the eigenvector of $\mathbf{A}$ regarding the eigenvalue λ_i.