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Synchronization of stochastic coupled systems via feedback control based on discrete-time state observations

Yongbao Wu, Bingdao Chen, Wenxue Li *

Department of Mathematics, Harbin Institute of Technology (Weihai), Weihai, 264209, PR China

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ABSTRACT

In this paper, we formulate and investigate the synchronization of stochastic coupled systems via feedback control based on discrete-time state observations (SCSFD). The discrete-time state feedback control is used in the drift parts of response system. Combining Lyapunov method with graph theory, the upper bound of duration between two consecutive state observations is provided. And a global Lyapunov function of SCSFD is presented, which derives some sufficient criteria to guarantee the synchronization of drive–response systems in the sense of mean-square asymptotical synchronization. In addition, the theoretical results are applied to stochastic coupled oscillators and second-order Kuramoto oscillators. Finally, two numerical examples are given to verify the effectiveness of the theoretical results.

1. Introduction

In recent years, considerable attention has been devoted to the study of coupled systems due to their extensive applications in many fields including mathematical [1,2], physics [3,4], biology [5–7]. Therein, it is noteworthy that a large quantity of coupled systems are discussed in a determinate case in relevant literatures [2,3,8,9]. However, in real applications, dynamical behaviors of coupled systems are affected inevitably by stochastic interference which exists in external environment. Hence, it is necessary to take random disturbance into account in the research of coupled systems. In fact, there are many papers related to stochastic coupled systems which mainly investigated the dynamic properties of stochastic coupled systems such as stability [1,10–12], synchronization [9,13–17], oscillatory property [18] and other dynamic properties [19,20].

As one of most important dynamical behaviors, synchronization indicates that the dynamical behaviors of coupled systems achieve the same time spatial state. For example, when electronic systems are synchronized, events at points far apart appear simultaneous or near-simultaneous from a certain perspective [21]. The investigation on the synchronization not only shows the principle of the synchronization but also can obtain the theoretical bases to take advantage of this character for benefiting human beings. Synchronization has many practical applications in real life. For instance, Hu et al. [22] proposed a regional traffic signal synchronization strategy to alleviate traffic congestion and to improve traffic efficiency in the network. Karimi-Ghartemani et al. [23] presented a new synchronization method which employs an enhanced phase-locked loop system to solve the problem of power electronic converters. Pursley et al. [24] investigated the synchronization of ovulation in dairy cows using PGF2alpha and GnRH. Above facts in life reveal the ubiquity of synchronization. Hence, it is necessary to study the synchronization of stochastic coupled systems.

* Corresponding author. Fax: +86 0631 5687660.
E-mail address: wenxue810823@163.com (W. Li).

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In order to make the stochastic coupled systems synchronized, we should impose controllers on them in most cases. In applications, there are many types of concepts of control such as adaptive control [25–27], optimal control [28,29], fuzzy control [30], observer-based control [31], feedback control [32–36] and so on. Among these controls, feedback control as a simple strategy can be effectively applied to synchronizing stochastic coupled systems due to its reasonable process of taking control by comparing the input value with output value of systems. There are two kinds of observation states in which feedback control is based on, continuous-time state and discrete-time state. We find that most existing literature of feedback control are based on continuous-time state [32–34]. However, discrete-time state observations are more practical than continuous-time state observations. For example, relevant data is collected by meteorological station through discrete-time state observation in weather prediction. Compared with data which comes from continuous-time, these data can also lead to a good result, moreover, meteorological station hardware requirement is low. Therefore, it is more efficient and costs less in practice for observing state only at discrete times. On account of these merits, investigating discrete-time state observations is meaningful and several results related to this issue are provided in [36–41].

Recently, there have been many results on synchronization and feedback control based on discrete-time state observations in literature. For instance, in [9], the reliable phase synchronization problem between two coupled chaotic fractional order systems was studied by constructing an active nonlinear feedback control scheme. A new approach to synchronization analysis of linearly coupled ordinary differential systems was derived based on geometrical analysis of the synchronization manifold by Lu et al. in [13]. Mao et al. [37] discussed the mean-square exponential stabilization of continuous-time hybrid stochastic differential equations by feedback controls based on discrete-time state observations. In [38], Pages et al. solved nonlinear filtering problems associated with discrete-time or continuous-time state processes and discrete-time observations by an optimal quantization approach. From the examples cited above, we learn that synchronization is practical and the feedback control based on discrete-time state observations has great potentialities. However, to the best of our knowledge, there is no result in literature investigating synchronization of stochastic coupled systems via feedback control based on discrete-time state observations (SCSFD). Hence, we will try to study the synchronization of SCSFD .

In the automatic control theory, Lyapunov method is a powerful tool to study synchronization. However, it is a challenging work to construct an appropriate Lyapunov function for stochastic systems. In [1], Li et al. developed a systematic approach that allows one to construct global Lyapunov functions for large-scale coupled systems by building blocks of individual vertex systems to solve this problem. Motivated by above analysis, we will study the synchronization of SCSFD by using Lyapunov method and graph theory in this paper. To ensure drive–response systems in the sense of mean-square asymptotical synchronization, we establish some sufficient conditions. After that, we apply theoretic results to stochastic coupled oscillators and second-order Kuramoto oscillators. Finally, two numerical examples will be given. The main contributions of the paper are highlighted as follows:

1. We use feedback control based on discrete-time state observations to investigate synchronization of stochastic coupled systems and get the upper bound of the duration between two consecutive state observations.

2. We construct global Lyapunov functions for SCSFD via building vertex-Lyapunov function of each vertex system, which avoids finding an appropriate Lyapunov function directly. Moreover, some sufficient criteria are obtained for the drive–response systems in the sense of mean-square asymptotical synchronization.

The remainder of this paper is outlined as follows. In Section 2, we present a mathematical model of SCSFD and give some preliminaries. In Section 3, some sufficient criteria for the sense of mean-square asymptotical synchronization of drive–response systems are provided. In Section 4, we employ theoretical results in stochastic coupled oscillators and second-order Kuramoto oscillators. Finally, two numerical examples are provided in Section 5.

2. Preliminaries and model formulation

2.1. Preliminaries

We introduce some notations firstly. Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a complete probability space with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). And $B(t)$ is a one-dimensional Brownian motion defined on the probability space. The mathematical expectation with respect to the given probability measure $\mathbb{P}$ is denoted by $\mathbb{E}(\cdot)$ and $\mathbb{R}^n$ stands for n-dimensional Euclidean space. The superscript "T" stands for the transpose of a vector or matrix. If $x \in \mathbb{R}^n$, then $|x|$ is Euclidean norm. The notations $\mathbb{L} = \{1, 2, \ldots, N\}$, $\mathbb{Z}^+ = \{1, 2, \ldots\}$, $\mathbb{R}^+ = [0, +\infty)$ and $m = \sum_{k=1}^{N} m_k$ for $m_k \in \mathbb{Z}^+$ are used. The $C^{2,1}(\mathbb{R}^n \times \mathbb{R}^+; \mathbb{R}^+)$ represents the family of all nonnegative functions $V(x, t)$ on $\mathbb{R}^n \times \mathbb{R}^+$ which are continuously twice differentiable in x and once in t. A digraph $\mathcal{G}$ is weighted if each arc (i, j) is assigned a positive weight a_{ij} and we call $A = (a_{ij})_{N\times N}$ as the weight matrix. And P is the Laplacian matrix of $(\mathcal{G}, A)$, in which P is defined as $P = (p_{kh})_{N\times N}$, where $p_{kh} = -a_{kh}$ for $k \neq h$ and $p_{kh} = \sum_{j\neq k} a_{kj}$ for $k = h$.

A useful lemma, which can be easily found in literature [1], is as follows:

Lemma 1 ([1]). *Suppose that $N \geq 2$. Let $c^{(k)}$ denote the cofactor of the* kth *diagonal element of P which stands for the Laplacian matrix of $(\mathcal{G}, A)$. Then the following identity holds:*

$$\sum_{k,h=1}^{N} c^{(k)} a_{kh} F_{kh}(x_k, x_h) = \sum_{\mathcal{Q}\in\mathbb{Q}} W(\mathcal{Q}) \sum_{(s,r)\in E(\mathcal{C}_{\mathcal{Q}})} F_{rs}(x_r, x_s),$$