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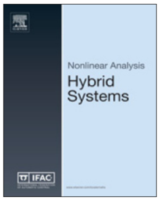

Observer synthesis for Linear Hybrid Systems with constrained discrete dynamics

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ABSTRACT

A Linear Hybrid System (*LHS*) is defined as a collection of Linear Systems (*LS*'s) and a switching signal determining, at each time, the *LS* structure that rules the behaviour of the *LHS*. These systems are being used to successfully represent different kinds of engineering systems. This work proposes an observer scheme for Linear Hybrid Systems (*LHS*'s) where the switching signal is unknown but it can be represented by a discrete event machine like automata, Petri nets, etc. The proposed observer structure takes advantage of the general observability characterization presented in Vázquez et al. (2015), where neither the observability of the discrete event machine nor the observabilities of the linear systems are required. First, the observer scheme estimates the discrete location, which is also named discrete state, by combining information from the continuous and discrete outputs. Next, the observer structure estimates the continuous state based on the knowledge of the visited locations, i.e., the discrete state trajectory. The observer scheme is flexible, allowing the use of different types of observers for the continuous and discrete states.

1. Introduction

A Linear Hybrid System (*LHS*) can be defined as a collection of Linear Systems (*LS*'s) and a switching signal determining, at each time, the *LS* structure that rules the behaviour of the *LHS*. These systems are being used to successfully represent different kinds of systems. For instance, in electrical power systems, the different operation modes (normal and different faulty behaviours) are modelled as a family of linear systems switching among them; in power electronics, switched power suppliers exhibit different operation modes that may be represented as a *LHS*; in process systems and other nonlinear systems (*NLS*), their state spaces are frequently partitioned such that the behaviour of a system at each partition is approximated by a linear system, leading to a *LHS*.

The study of the fundamental properties of *LHS*'s has received great attention during the last years. In particular, the observability property allows to estimate the switching signal and the continuous state by using the knowledge of the system's input and output signals. This property has been broadly studied due its importance in different real applications. For instance, the knowledge of the switching signal and the continuous state allows to detect faults and their magnitude in electrical power systems; to determine the most appropriated linear representation in *NLS* approximated by *LHS*'s, among many other applications.

The observability in *LHS*'s has been studied assuming different hypotheses. For instance, in [1] and [2] the switching signal is designed to improve the observability. On the other hand, in [3–5] it is assumed that the switching signal is known, in this

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setting the continuous state of the *LHS* is estimated by the combination of partial observations obtained when visiting each *LS*. In [6–9] it is considered that the switching signal is unknown, allowing the switching from any *LS* to any other *LS*. In [10] necessary conditions for the observability of piece-wise affine hybrid system were presented, in which the switching signal is unknown but the switching is state dependent. Without any information about the switching sequence, the conditions required for observability are quite restrictive. In general, it is required that each *LS* is *observable* and each pair of *LS*'s is *distinguishable* [6,11,12], i.e., two different *LS*'s must not produce the same output trajectory in order to be able to determine the discrete location from the information provided by the continuous output. For this case, different works have addressed the synthesis of the location observer, for instance, [13–15].

Most of the observability and observer synthesis works consider that the *LHS* can switch from any *LS* to any other *LS*, however there are interesting cases where such switching assumption does not hold. For instance, in process systems it frequently occurs that discrete actuators, leading to operation modes (i.e., *LS*'s), must be activated according to certain sequence, see for instance, [16–18]. Moreover, nonlinear models are frequently approximated by *LS*'s operating at different operation points, leading to autonomous switchings as a piecewise affine system, where a *LS* can only switch to its neighbours, see for instance, [19,20]. In power electronic systems, it is frequently found that some semiconductors are controlled but others are autonomously driven, see for instance [21,22]. Thus, switchings from some *LS*'s operation modes to others may be impossible to occur or must be avoided for stability reasons. In traffic systems in urban areas, when considering a fluid flow approach for the traffic behaviour, traffic lights lead to hybrid models (for instance, [23–26]), in which the discrete modes, induced by traffic lights, evolve according to a predefined sequence. In chaos-based secure communications in which the information is modulated by using a chaotic attractor generated by a *LHS*, the demodulation problem involves an observer synthesis in which the knowledge on the switching structure can be used [27].

There are few works in the literature addressing the case in which the switching is constrained. In [28] there is defined a hybrid system of which the switching signal is generated by a Petri net (*PN*), in which some places and transitions are measurable. The goal there is to developed an observer for the discrete state and for the continuous state. Nevertheless, in [28] it is assumed that the continuous systems are observable. Moreover, the *PN* must be observable after each switching just by using the discrete measurements. In fact, all the events are distinguishable from each other. Closely related works have been reported in [29,30]. In [29] the observer synthesis problem is addressed considering that the switching signal is produced by a finite automaton. There, the discrete location is determined by computing residuals from Luenberger observers. Thus, a discrete observer is proposed as a finite automaton, whose states are defined as sets of currently possible locations (explanations), leading to an exponential number of states in the discrete observer. In [30], the observability of *LHS* where the discrete dynamic is determined by a Moore automaton is addressed. In this setting, each pair of states with the same discrete output is required to be distinguishable, which requires their associate *LS* to be observable. Such conditions can be relaxed in the *eventual observability* case [31], where neither the observability of the continuous states nor the observability of the discrete system is required. Furthermore, the observer synthesis problem was not addressed in [30].

The work introduced here is based on the *eventual observability* analysis presented in [31]. In that work, it was found that the information provided by the continuous output can be used together with the information provided by discrete sensors and the structure of the underlying discrete event system, but not its state, to estimate both the discrete and continuous states of a *LHS*, greatly relaxing the observability conditions. In fact, the observability of each *LS* is not a necessary condition for eventually determining, after a certain number of switchings, which is the evolving *LS*. Furthermore, the observability of the underlying discrete event system, by only using discrete measurements, is not required.

The contribution of this work is the introduction of an observer scheme for eventually observable *LHS*'s. Our approach uses a *PN* for capturing how the *LS*'s switch with each other. In our observer scheme, a couple of algorithms are firstly used to combine the information from the continuous and the discrete outputs for the estimation of the discrete location. By using such information, a Petri net observer estimates the discrete location, i.e., the marking of the *PN*. Next, based on the knowledge of the visited locations, i.e., the *PN*'s marking evolution, a continuous observer estimates the continuous state. The conditions on the *LHS* required for our observer scheme are more relaxed than those required in [28], as mentioned in the previous paragraph. Comparing to [29,30], the use of a *PN* allows to have a compact representation for the discrete observer. Furthermore, we distinguish between the information from the continuous state that is useful to determine for the first time the current state and the information useful for determining the location after the first estimation, obtaining thus more relaxed conditions for observability.

This paper is organized as follows. In Section 2 some basic definitions and results about *LHS*'s and *PN*'s are provided. In Section 3, some observability results are recalled from [31], explaining how to translate the modal information of the *LS*, that may help to distinguish between them, into the *PN* framework. The observer scheme is introduced in Section 4. Such observer scheme estimates both the discrete state and the continuous state of the *LHS*. The application of the observer is illustrated in Section 5. Finally, some conclusions and future work are presented in Section 6.

2. Basic concepts and definitions

2.1. Linear hybrid systems

Definition 1. A Linear Hybrid System (*LHS*) $\Sigma_{\alpha(\tau)}$ is a collection of linear systems (*LS*'s) $\mathcal{F} = \{\Sigma_1, \dots, \Sigma_m\}$, each one defined in the state space $\mathcal{X} = \mathbb{R}^n$, and a switching signal $\alpha(\tau)$, taking values in $\{1, \dots, m\}$, that determines the evolving linear