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# Distributed adaptive switching consensus control of heterogeneous multi-agent systems with switched leader dynamics

Chengzhi Yuan

*Department of Mechanical, Industrial and Systems Engineering, University of Rhode Island, Kingston, RI 02881, United States*

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ABSTRACT

In this paper, the leader-following distributed consensus control problem is addressed for general linear multi-agent systems with heterogeneous uncertain agent dynamics and switched leader dynamics. Different from most existing results with a single linear time-invariant (LTI) leader dynamics, the leader dynamics under consideration is composed by a family of LTI models and a switching logic governing the switches among them, which is capable of generating more diverse and sophisticated reference signals to accommodate more complicated consensus control design tasks. A novel distributed adaptive switching consensus protocol is developed by incorporating the model reference adaptive control mechanism and arbitrary switching control technique, which can be synthesized by following a two-layer hierarchical design scheme. A numerical example has been used to demonstrate the effectiveness of the proposed approach.



## 1. Introduction

Over the past two decades, the problem of multi-agent consensus has been extensively studied in the controls community. As one of the most important and fundamental cooperative control objectives, consensus of multi-agent systems specifies the control objective of driving all the agents to reach a certain common agreement through inter-agent coordinations. The idea behind consensus provides a fundamental principle for the design of distributed multi-agent coordination algorithms, which have found such wide applications as formation control, distributed estimation/filtering, and distributed optimization. Along this research topic, numerous interesting results on theoretical algorithm developments [1–5] and real engineering applications [6–9] have been reported in the literature. Comprehensive overview and summary on this topic can be found in the surveys [10–12] and books [13–15], among others. Despite of the vast amount of literature in the field, there still exist many important yet challenging issues that have not been fully addressed, two of which are particularly worth to be mentioned.

The first issue pertains to the heterogeneity and uncertainty of agent dynamics. Specifically, majority of the existing results have focused on distributed consensus control of identical agents (see, e.g., [16–18]); while in many real applications, multi-agent systems may consist of heterogeneous agents due to various reasons. To overcome this issue, some attempts have been made recently to develop new distributed cooperative control techniques for heterogeneous multi-agent systems. In particular, motivated by the classical output regulation theory [19,20], Su and Huang proposed a cooperative output regulation approach for leader-following output synchronization of heterogeneous multi-agent systems [21–24]. However, when concerning about the consensus control problem, potential limitations of the cooperative output regulation scheme

*E-mail address:* cyuan@uri.edu.

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lie in that it typically requires each agent's control input number is at least equal to the state number in order to ensure solvability of the associated output regulator equations, which could be infeasible in many practical situations. As an alternative, distributed adaptive control techniques provide a promising solution to the heterogeneity and uncertainty issue in multi-agent consensus control. For example, for a class of heterogeneous uncertain multi-agent systems with general linear agent dynamics, the model reference adaptive control approach was utilized in [25] to achieve robust consensus performance. This work was subsequently extended to nonlinear uncertain multi-agent systems by using artificial neural networks [26]. It has also been demonstrated in many other works [27–29] that adaptation mechanism played an important role in dealing with various uncertainties involved in multi-agent coordinations.

The second issue relates to the leader-following consensus control framework [17,24]. In this framework, a (virtual) leader is assigned to guide the whole group to reach state consensus through a distributed protocol. Typically, the final consensus agreement among the agents is pre-specified by the leader's state, which implies that more complicated behaviors of the leader will lead to more diverse and sophisticated consensus behaviors for the followers. However, it is surprising to notice that virtually all of the existing leader-following distributed consensus control techniques assume that the leader agent obeys a single neutrally stable linear time-invariant (LTI) dynamics, which would largely limit the leader's dynamical behaviors to simple constant or periodic signals. In many practical situations, a purely constant or periodic signal many not be satisfactorily used to fulfill the desired multi-agent coordination tasks. Therefore, it is important and meaningful to consider more complex leader dynamics such that more diverse and sophisticated reference signals can be realized to accommodate more complicated multi-agent coordination tasks and thus enhance the multi-agent systems' capability.

In this paper, to overcome the above-mentioned deficiencies of existing distributed consensus control methods, we will propose a new distributed control technique for leader-following consensus of heterogeneous multi-agent systems with general linear uncertain agent dynamics and a switching-type nonlinear leader dynamics. The nonlinear leader dynamics is composed by a family of LTI models and a switching rule governing the switches among them, which is capable of generating more diverse and complex reference signals that cannot be obtained by using a pure LTI model. This will thus significantly enrich the multi-agent consensus behaviors. Moreover, the introduction of a switching logic will further increase the distributed control system design flexibility, as well as the operational capability of multi-agent systems in real applications. Nevertheless, considering a switched leader dynamics will also complicate the associated multi-agent system's stability analysis and control design problems. Even under the setting of single dynamical systems, the problems of switching stability and stabilization are well-known challenging, as many conventional linear/nonlinear system theories might not be applicable for switched systems due to the discontinuity and non-smoothness of the associated vector fields. Although effective techniques and tools have been developed in recent years for switching stability analysis and switched control designs (such as the common Lyapunov function technique [30], the multiple Lyapunov function techniques [31], min-switching logic [32], (average) dwell-time switching logic [33], hysteresis switching logic [34], and so on), it is still an open problem of how to incorporate these techniques in multi-agent distributed control designs.

The contributions of this paper can be understood as follows. A novel distributed adaptive switching (DAS) consensus protocol is first proposed, which consists of two portions: a DAS observer and a DAS controller. Specifically, the DAS observer part will be used to online estimate the leader's information for each agent via information sharing with their neighbors; while the DAS controller part will be responsible for stabilizing the overall network and achieving consensus performance by utilizing agent's own system information (i.e., without inter-agent information sharing). Such a distributed controller structure facilitates the DAS consensus protocol design in a hierarchical fashion. More importantly, it enables incorporation of many classical stability analysis and control design techniques into the multi-agent distributed control designs. In particular, the distributed adaptive switching stability and parameter convergence issues will be addressed in this paper by merging the model reference adaptive control (MRAC) technique [35] and the common Lyapunov function technique from the switching control context [30]. A numerical example will be used to demonstrate the effectiveness of the proposed approach.

The rest of the paper is organized as follows. Some preliminary reviews on graph theory and the problem statement are given in Section 2. The main results of this paper including the DAS observer and DAS controller designs are presented in Section 3. Section 4 provides numerical studies to illustrate the effectiveness of the proposed control design approach. Conclusions are drawn in Section 5.

## 2. Preliminaries and problem statement

### 2.1. Notation and graph theory

Throughout the paper, we use $\mathbb{R}$ to denote the set of real numbers. $\mathbb{R}_+$ stands for the set of positive real numbers. $\mathbb{R}^{m\times n}$ is the set of real $m\times n$ matrices, and $\mathbb{R}^n$ represents the set of real $n\times 1$ vectors. The identity matrix of dimension $n\times n$ is denoted by $I_n$. $\mathbf{1}_n$ denotes an $n$-dimensional column vector with all elements being 1. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ are used to denote the sets of real symmetric $n\times n$ matrices and positive definite matrices, respectively. A block diagonal matrix with matrices $X_1, X_2, \ldots, X_p$ on its main diagonal is denoted by $diag\{X_1, X_2, \ldots, X_p\}$. The notation $A\otimes B$ stands for the Kronecker product of matrices $A$ and $B$. For a matrix $A$, $\vec{A}$ is the vectorization of $A$, obtained by stacking the columns of the $A$ on top of one another. For a series of column vectors $x_1, \ldots, x_n$, $col\{x_1, \ldots, x_n\}$ represents a column vector by stacking them together. For two integers $k_1 < k_2$, we denote $\mathbf{I}[k_1, k_2] = \{k_1, k_1 + 1, \ldots, k_2\}$. For a matrix $M \in \mathbb{C}^{m\times n}$, $M^T$ denotes its transpose. For $x \in \mathbb{R}^n$, its norm