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New criteria for exponential stability of switched time-varying systems with delays and nonlinear disturbances

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ABSTRACT

In this paper, we study exponential stability of switched time-varying systems with bounded delays and nonlinear disturbances. By introducing a model transformation and using a novel method which does not involve the Lyapunov–Krasovskii functional, new explicit criteria for exponential stability of the system under arbitrary switching have been established in terms of Metzler matrices. Numerical examples show that the obtained results can be applied to some cases not covered by preceding results.

1. Introduction

Up until now, stability of switched systems has attracted the attention of many researchers, due to its wide applications in traffic control, chemical processing, network control, converter applications and so on. Asymptotic stability and stabilization play a key role in the theory of switched systems, and have been widely investigated in recent years. To mention a few, we refer the reader to monographs [1–3], survey papers [4–6], and some excellent papers [7–15].

For switched linear systems, one important problem is asymptotic stability under arbitrary switching. Although it has been proved that asymptotic stability of switched linear systems under arbitrary switching is equivalent to the existence of a common Lyapunov functional [1], it is very hard to construct a common Lyapunov functional of the general form which can be easily verified. As a result, common quadratic Lyapunov functionals are usually used to study asymptotic stability of switched linear systems under arbitrary switching [4]. So far, the existence of common quadratic Lyapunov functionals is still unsolved except in some special cases [16].

Motivated by positive systems whose state variables take only nonnegative values, a common linear copositive Lyapunov functional method was used to study asymptotic stability of positive switched linear systems under arbitrary switching [17–21]. Unlike common quadratic Lyapunov functionals, the existence of common linear copositive Lyapunov functionals has been solved thoroughly in [22–24]. Recently, a method which is different from the common linear copositive Lyapunov functional method was applied to derive delay-independent criteria for asymptotic stability of positive switched linear delay systems under arbitrary switching in [25–27]. When the involved switched system is not necessarily positive, by using a method different from that in [26,27], delay-independent criteria for asymptotic stability of the system under arbitrary

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switching were presented in [28]. However, the involved system matrices were enlarged directly into Metzler matrices and nonnegative matrices, which leads to conservativeness inevitably.

Note that all of the aforementioned works have been mainly focused on switched linear systems with constant matrices. For stability of switched time-varying systems, the developed methodologies such as Lyapunov–Krasovskii functional cannot be used because they either lead to matrix Riccati differential equations or indefinite LMIs. On the other hand, time delay has attracted much attention due to its common presence in practical engineering and its detrimental effects on performance of systems such as oscillation [29–31]. Therefore, in this paper we will focus on deriving new delay-dependent criteria for exponential stability of switched time-varying systems under arbitrary switching. Both time-varying delays and nonlinear disturbances have been taken into consideration. By using a model transformation and a novel approach which is different from the Lyapunov–Krasovskii functional method, explicit delay-dependent criteria for exponential stability of the system under arbitrary switching have been established in terms of Metzler matrices, which can be applied to some cases not covered by existing results in the literature.

The paper unfolds as follows. In Section 2 we present the notation used through the paper as well as preliminaries for our results. Section 3 then focuses on deriving sufficient conditions for exponential stability of switched time-varying systems with delays and nonlinear disturbances. In Section 4 we give two examples to illustrate the theoretical results of this paper. Finally, conclusions are drawn in Section 5.

2. Preliminaries

For a given positive integer m, denote $\langle m\rangle = \{1, 2, \dots, m\}$. $\mathbb{R}^n$ is the n-dimensional real vector space with the norm $\|x\| = \max_{i\in\langle n\rangle}\{|x_i|\}$, where $x = (x_i) \in \mathbb{R}^n$. $\mathbb{R}^{n\times n}$ is the set of real $n \times n$-matrices. I_n denotes the n-dimensional identity matrix. Say a vector or matrix $A \succ 0$ ($\succeq 0$, $\preceq 0$, $\prec 0$) if all its components are positive (nonnegative, nonpositive, negative). For $A, B \in \mathbb{R}^{n\times n}$, denote $A \preceq B$ if $A - B \preceq 0$. A Metzler matrix is a square matrix whose off-diagonal entries are nonnegative. A square matrix is *Hurwitz* if the real part of each of its eigenvalues is negative. Given $x = (x_i) \in \mathbb{R}^n$ and $A = (a_{ij}) \in \mathbb{R}^{n\times n}$, denote $|x| = (|x_i|)$, $|A| = (|a_{ij}|)$ and $|A|_* = (a^*_{ij})$ with $a^*_{ii} = a_{ii}$ and $a^*_{ij} = |a_{ij}|$ for $i \neq j$, $i, j \in \langle n\rangle$. Given a series of matrices $\{A_k = (a^{(k)}_{ij}) \in \mathbb{R}^{n\times n} : k \in \langle m\rangle\}$, denote $A_{\max} = (a^{\max}_{ij})$ with $a^{\max}_{ij} = \max_{k\in\langle m\rangle}\{|a^{(k)}_{ij}|\}$ for $i, j \in \langle n\rangle$. Denote the ith row of a matrix A by $\mathrm{row}_i(A)$ for $i \in \langle n\rangle$.

Consider the following continuous-time switched time-varying system subject to delay and nonlinear disturbances

$$\dot{x}(t) = A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)x(t - \tau(t)) + f_{\sigma(t)}(t, x(t), x(t - \tau(t))), \quad t \geq 0, \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state vector, the piecewise continuous function $\sigma : [0, \infty) \to \langle m\rangle$ is the switching signal that specifies, at each time, the index of the active system, $A_k(t) = (a^{(k)}_{ij}(t)) \in \mathbb{R}^{n\times n}$, $B_k(t) = (b^{(k)}_{ij}(t)) \in \mathbb{R}^{n\times n}$, $k \in \langle m\rangle$, are continuous matrix functions, $f_k(t, x, y) : [0, \infty)\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function satisfying local Lipschitz condition with respect to x and y, $f_k(t, 0, 0) = 0$ for $t \geq 0$, $\tau(t) : [0, \infty) \to [0, \infty)$ is the time delay which is continuous and satisfies $0 \leq \tau(t) \leq h$, $h > 0$ is a constant. Assume throughout this paper that $x(t) \equiv x(0)$ when $t \leq 0$, where $x(0)$ is the initial condition.

We also consider the following switched time-varying system with distributed time delay and nonlinear disturbances

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}(t)x(t) + B_{\sigma(t)}(t)x(t - \tau(t)) + C_{\sigma(t)}(t)\int_{t-\rho(t)}^{t} x(s)ds \\ &\quad + f_{\sigma(t)}\left(t, x(t), x(t - \tau(t)), \int_{t-\rho(t)}^{t} x(s)ds\right), \quad t \geq 0,\end{aligned} \tag{2.2}$$

where x, $A_k(t)$, $B_k(t)$, $\sigma(t)$ and $\tau(t)$ are defined as above, $C_k(t) = (c^{(k)}_{ij}(t)) \in \mathbb{R}^{n\times n}$, $k \in \langle m\rangle$, are continuous matrix functions, $f_k(t, x, y, z) : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function satisfying local Lipschitz condition with respect to x, y and z, $f_k(t, 0, 0, 0) = 0$ for $t \geq 0$, $\rho : [0, \infty) \to [0, \infty)$ is the distributed time delay which is continuous and satisfies $0 \leq \rho(t) \leq h$, $h > 0$.

For the particular case when $A_k(t) \equiv \bar{A}_k$, $B_k(t) \equiv \bar{B}_k$, $C_k(t) \equiv \bar{C}_k$ and $f_k \equiv 0$, where $\bar{A}_k$, $\bar{B}_k$ and $\bar{C}_k$ are $n\times n$ constant matrices for $k \in \langle m\rangle$, system (2.1) and system (2.2) reduce to the following simpler forms

$$\dot{x}(t) = \bar{A}_{\sigma(t)}x(t) + \bar{B}_{\sigma(t)}x(t - \tau(t)), \quad t \geq 0, \tag{2.3}$$

and

$$\dot{x}(t) = \bar{A}_{\sigma(t)}x(t) + \bar{B}_{\sigma(t)}x(t - \tau(t)) + \bar{C}_{\sigma(t)}\int_{t-\rho(t)}^{t} x(s)ds, \quad t \geq 0. \tag{2.4}$$

Under assumptions that $\bar{A}_k$ is a Metzler matrix and $\bar{B}_k$ is a nonnegative matrix for $k \in \langle m\rangle$, it was proved in [27] that system (2.3) is asymptotically stable under arbitrary switching if there is an n-dimensional vector $\xi \succ 0$ such that $(\bar{A}_k + \bar{B}_k)\xi \prec 0$ for $k \in \langle m\rangle$. When A_k and B_k, $k \in \langle m\rangle$, are not necessarily Metzler matrices and nonnegative matrices, respectively, it was shown in [28] that system (2.3) is still asymptotically stable under arbitrary switching if there is an n-dimensional vector $\xi \succ 0$ such that $(|\bar{A}_k|_* + |\bar{B}_k|)\xi \prec 0$ for $k \in \langle m\rangle$.