Microelectronics Journal 45 (2014) 539–546

Contents lists available at ScienceDirect

Microelectronics Journal

journal homepage: www.elsevier.com/locate/mejo

Heat spreading in a thin longitudinal fin

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ARTICLE INFO

Article history:
Received 3 October 2012
Received in revised form
4 July 2013
Accepted 15 November 2013
Available online 17 December 2013

Keywords:
Heat spreading
Thin longitudinal fin
Plate heat sink
Line source
Characteristic length

ABSTRACT

In view of the trend towards higher power densities in ever shrinking geometries, understanding heat spreading fundamentals is gaining importance. In this paper heat spreading in thin longitudinal geometries is considered. This geometry is of practical interest in one-dimensional Cartesian geometries. A characteristic length is derived and it is shown that this has physical significance for the distance that heat spreads, and for the total amount of heat cooled away. Furthermore, it is investigated when "thin" is a viable assumption. The use of the characteristic length is illustrated for the case of a line source cooling to a plate and for the case of the fins of a plate heatsink. The results are compared to numerical simulations. The work is an extension of the authors' earlier work on heat spreading in infinite longitudinal geometries and heat spreading in infinite and finite circular geometries.

1. Introduction

Simultaneous advances in miniaturization and performance are a continuing trend in the development of new generations of electronic products. Therefore, source power densities are increasing while at the same time allowable source temperatures stay the same or are lowered. To cool small sources to realistic temperatures, and obtain realistic power densities at the component and system level, heat spreading is gaining in importance.

The thin longitudinal geometry is an important form factor in heat spreading. It covers spreading along the fin height in pin fin and plate fin geometries, spreading along a line in narrow strip geometries, and heat spreading from line sources to a thin plate.

Heat spreading in the thin infinite longitudinal case was addressed previously in [1], and radial heat spreading from a small heat source on a thin plate has been addressed in [2].

The present paper addresses the derivation of the characteristic length for the longitudinal (Cartesian) case in Section 2, and investigates the physical meaning of this length in Section 3. In Section 3 also approximations are derived for the temperature distribution along the fin and for the amount of heat transferred to the ambient. In Section 4, criteria are given as to under what circumstances a fin can be considered thin. Sections 5 and 6 show an application example for the case of a strip heat source on a thin plate and for the case of a plate heatsink, and compare to numerical results.

The results demonstrate the engineering relevance of the derived characteristic length based approximation in enabling quick engineering estimations.

2. Derivation of length scale

In [1] the characteristic length for the infinitely long, thin longitudinal case was derived directly from the differential equation. An alternative approach was taken in [2], inspired by the work of Adrian Bejan [3,4]. A similar approach is now demonstrated for the longitudinal case.

One of the guiding principles in the constructal approach is that relevant length scales appear when competing physical effects are of the same magnitude. In heat spreading, the competing mechanisms are the conductive heat spreading in the thin plate and the convective heat transfer from the thin plate to the ambient.

Consider the case depicted in Fig. 1. A thin flat plate of thickness t (m), width w (m) is heated at the left side surface at $x=0$. The far right surface at the end is adiabatic. The thermal conductivity is k $\mathrm{W m^{-1} K^{-1}}$. Cooling is by means of heat transfer coefficient h $\mathrm{W m^{-2} K^{-1}}$ on one side. The heating and cooling are uniform over the width direction of the plate and the plate is so thin that the temperature is uniform over the thickness.[1] The plate's temperature distribution is a function of the coordinate x along the length direction. The thermal resistances for conductive and for convective heat transfer of a fin section of length L are

$$R_{conduction} = \frac{L}{twk} \tag{1}$$

$$R_{convection} = \frac{1}{Lwh} \tag{2}$$

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[1] Relevant criteria as to "thinness" are found in Section 4.

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http://dx.doi.org/10.1016/j.mejo.2013.11.010

Nomenclature

Bi	Biot number (dimensionless)
h	heat transfer coefficient ($\mathrm{W m^{-2} K^{-1}}$)
k	thermal conductivity ($\mathrm{W m^{-1} K^{-1}}$)
L	length (m)
L_c	characteristic length (m)
L_{source}	length of source (m)
q	transferred heat (W)
Q	scaled transferred heat (dimensionless)
R	thermal resistance (K/W)
T	temperature (C)
t	thickness (m)
w	width (m)
x	distance along the length (m)
η	scaled length distance (Eq. (4)) (dimensionless)
θ	scaled temperature (Eq. (4)) (dimensionless)
0	(subscript) at $x=0$
e	(subscript) at $x=L$

Equating the resistances leads to

$$L^2 = \frac{tk}{h}$$

$$L = \sqrt{\frac{tk}{h}} \equiv L_c \tag{3}$$

Expression (3) is the same L_c expression as derived from the governing differential equations in [2].

3. What does the length scale mean?

We now define a scaled length η and scaled temperature θ, which are used to investigate the physical meaning of L_c.

$$\eta \equiv \frac{x}{L_c}$$

$$\theta \equiv \frac{T - T_{ambient}}{T_0 - T_{ambient}} \tag{4}$$

Furthermore we define L as the total length of the fin, $\theta_0 = 1$ as the scaled temperature at $x=0$, the edge of the source, and θ_e as the scaled temperature at the fin tip $x=L$ or in scaled variables $\eta = \eta_e = L/L_c$.

The total heat transferred is given by

$$q = \int_0^L wh(T - T_{ambient})dx = \int_0^{\eta_e} wh\theta(T_0 - T_{ambient})L_c d\eta \tag{5}$$

This scales as

$$Q \equiv \frac{q}{whL_c(T_0 - T_{ambient})} = \int_0^{\eta_e} \theta d\eta \tag{6}$$

Note that (6) demonstrates that Q equals the area below the $\theta(\eta)$ curve. Furthermore it follows from (6) that

$$R_{\ edge-ambient} \equiv \frac{T_0 - T_{ambient}}{q} = \frac{1}{QwhL_c} \tag{7}$$

The temperature field in the geometry is well known [5]. In dimensionless form, the scaled temperature, $\theta(\eta)$, relates to the scaled distance, η, and the length of the fin, η_e as

$$\theta(\eta) = \frac{\cosh(\eta_e - \eta)}{\cosh(\eta_e)} = \cosh(\eta) - \tanh(\eta_e)\sinh(\eta) \tag{8}$$

3.1. Infinite case

For long fins, $\eta_e > 2$, tan h$(\eta_e) \approx 1$ and

$$\theta = \cos h(\eta) - \sin h(\eta) = e^{-\eta} \tag{9}$$

The temperature decays exponentially with rate 1 from $\theta = 1$ at $\eta = 0$, to $\theta = 0$. In exponential decay, the influence of the imposed end temperature continues indefinitely but it diminishes fast: At $\eta = 1$, $\theta = 0.37$; at $\eta = 2$, $\theta = 0.14$, and at $\eta = 3$, $\theta = 0.05$. In other words less than 5% of the imposed temperature is found beyond a distance of $3L_c$.

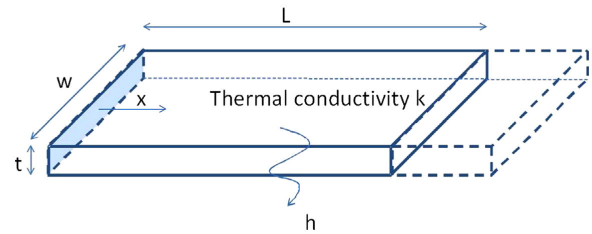

Fig. 1. Longitudinal fin geometry.

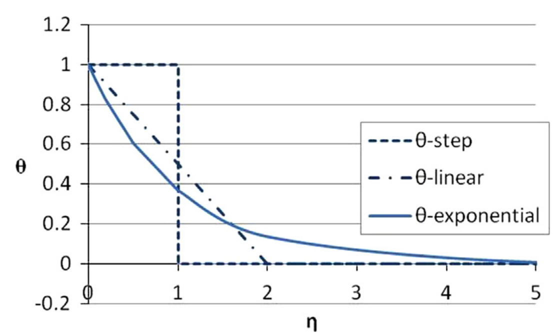

Fig. 2. Temperature distribution for the infinite fin.

The area below the exponential curve is finite and equal to 1. Eq. (6) shows that the scaled heat loss to the ambient, Q, is equal to this area. Thus

$$Q \equiv \frac{q}{whL_c(T_0 - T_{ambient})} = 1$$

$$q = whL_c(T_0 - T_{ambient}) \tag{10}$$

This shows that the infinitely long rectangular fin with T_0 imposed at the end $x=0$, experiences heat loss to the ambient as if a length L_c is heated to T_0, and the remainder of the fin stays cold. Graphically, this is represented by a step temperature distribution over distance L_c, as illustrated in Fig. 2. The step temperature distribution exactly matches the source temperature at $x=0$ and exactly matches Q, the total heat transferred to ambient, but is less representative of the true exponential temperature drop off for $0 < x < 3L_c$. A linear temperature drop over distance $2L_c$ has a much better fit: The source temperature at $x=0$ is matched, the area below the curve, Q, equals 1, so the total heat transferred to ambient is an exact match also. In addition the temperature is monotonically decreasing with the distance from the source, which is a better match to physics since heat flows from hot to cold. The exponential temperature decay, the step temperature distribution and the linear temperature drop are compared in Fig. 2. In all three cases, the source temperature is