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Optimal control of nonlinear switched system with mixed constraints and its parallel optimization algorithm

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ABSTRACT

We consider an optimal control problem for a class of nonlinear switched systems with mixed constraints, in which both the mode sequence of active subsystems and the number of switches are not prescribed. By introducing convex combinations of the subsystems and the constraints, an embedded system is formulated. The relationships between the trajectories of the switched system and the corresponding embedded system are discussed. Necessary optimality conditions and the existence condition of optimal solutions are derived. A constructive parallel algorithm is proposed for solving optimal or suboptimal solutions of the considered problem. In order to avoid the case that the switching times may be lost, the time-scaling transformation is applied to transform the variable switching times to pre-fixed switching times. Finally, an optimal control of fed-batch fermentation is solved by applying the proposed algorithm, and the numerical results show that the proposed algorithm is effective.

1. Introduction

Switched systems are a class of hybrid systems encountered in many practical areas, which involve switches among several subsystems depending on various factors. The earliest result was proposed by Witsenhausen [1], in which a maximum principle is derived for hybrid systems with autonomous switching only. Since then, the study of hybrid and switched systems has attracted growing interest from both theoretical and applied fields [2].

Generally, a switched system consists of a finite number of dynamical subsystems and a proper switching rule that describes which subsystem is active at each moment of time horizon. In each subsystem, which is also referred to as a mode, the evolution of the state is described by a set of differential or difference equations. As a special type of hybrid system, switched systems arose in a variety of applications, such as automobiles and locomotives with different gears [3], biological systems [4], manufacturing processes [5], DC–DC converters [6] and shrimp harvesting model [7]. In these problems, optimal control problem of switched systems is one of the most challenging and important problems, which involves simultaneously finding a mode sequence as well as the switching times and the continuous control inputs for each active mode such that a cost functional is optimized subject to some constraints on state and controls.

The problem of determining optimal control rules for switched systems has been widely investigated over the past years from both theoretical and computational points [8]. For continuous-time switched systems, most of the literature studied necessary and/or sufficient conditions for a trajectory to be optimized, with the introduction of new versions of

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the maximum principle [9–11]. The existences of optimal control for switched systems were also investigated [12,13]. Optimization techniques for some delayed switched systems were studied in Verriest et al. [14]. For finding an optimal solution, the methods based on dynamic programming [15–17], variational techniques such as the Maximum Principle [18–22], and direct differentiation of the cost functional [23–28] have been investigated.

It is worth mentioning that Xu and Antsaklis [23,29,30] considered the optimal control of a nonlinear switched system under a fixed, pre-specified modal sequence and proposed a bi-level hierarchical optimization algorithm. For the same problem, Shaikh and Caines [31] searched through all possible sequences within a fixed distance of the pre-specified sequence to find a sequence with a lower cost after performing the original optimization. Axelsson and Egerstedt et al. [26,32] considered a special case of nonlinear autonomous switched systems, and employed a similar bi-level hierarchical algorithm. At the lower level, they kept the modal sequence fixed and determined the optimal mode duration and optimal continuous input. At the higher level, they employed a single mode insertion technique to construct a new lower cost sequence. For the constrained nonlinear switched systems, Gonzalez et al. [27] constructed a similar bi-level hierarchical algorithm, and perfected the single mode insertion technique. Wardi et al. [33] developed a provably-convergent algorithm that eliminated the potential drawback of the single mode insertion technique while extending the update method of mode-sequence from single intervals and single mode-switching to general time-sets and multiple modes. The scheme for optimizing the mode sequence in the above literature can be summarized as follows: an initial mode sequence was given and then methods for varying the mode sequence to decrease the cost were used.

Control parametrization method [34] and time scaling transform methods [35] are important techniques in solving optimal control problems. In control parametrization scheme, the controls were approximated by a linear combination of basis functions, where the coefficients in the linear combination were decision variables to be chosen optimally. Time scaling transform method is a technique for transforming the optimization of variable switching times into pre-fixed switching times. Based on the combination of control parametrization and time scaling transform method, a switching optimal control technique was proposed by Li and Teo et al. [25] to find the approximate optimal control inputs and switching times, which have been used extensively [36–38]. For more details, see Refs. [39–41].

It should be pointed out that many methods mentioned above require some assumptions that the number of switches or even the mode sequence were pre-fixed. A more general class of switched systems is that both the number of switches and the mode sequence are decision variables to be optimized. Since the switching sequence is in essential discrete optimization variables, this class of problems leads to mixed-integer dynamic programming problems rather than classical dynamic programming problems. A large number of approaches have been explored to solve this class of problems, including heuristic approach [42], bi-level optimization [43] and binary relaxation approach [44], and so on. The disadvantage of the heuristic method, however, is that the switching times obtained via the transformation is not quite equivalent to the original optimal control problem. In the bi-level optimization approach, the inner level involves solving a switching optimal control problem with fixed mode sequence, while the outer level involves solving a discrete optimization problem which is generally solved by particle swarm optimization [45] or filled function method [43]. The binary relaxation approach constructs an equivalent optimization problem subject to additional linear and quadratic constraints which essentially restrict the feasible region to a disjoint set. Thus, standard gradient-based optimization methods will usually struggle with these constraints. An exact penalty method was introduced in [44] to solve this non-smooth optimal control problem, while standard optimization algorithms are generally not sufficient. Bengea and DeCarlo [18,46] considered an optimal control problem for bimodal switching system without imposing restrictions on the number of switches, in which the switched system was embedded into a larger family of nonlinear systems that can be directly handled by classical control theory. On the basis of the embedded framework, Wei et al. [47] applied the piecewise smooth approximation on states and controls to transform the switching optimal control problem to parameterized one, and traditional nonlinear programming techniques such as sequential quadratic programming (SQP) can be used instead of solving a mixed-integer programming problem. However, the variable number of switches was still not considered in their work. Vasudevan et al. [48,49] presented a conceptual algorithm and an implementable algorithm for the optimal control of constrained switched systems based on Polak's framework of optimality functions and consistent approximation methods in Polak [50]. Similar to the ideas described in [18], they considered a continuous-valued input by relaxing the discrete-valued input, and performed traditional optimal control. And then the continuous-valued input was projected back to a pure discrete-valued input by employing an extension to the classical chattering lemma.

Unlike the above frameworks, we consider a general optimal control problem involving a class of nonlinear switched systems composed of s ($s \geq 2$) dynamical subsystems subject to mixed state-control constraints, in which the switching sequence is assumed to be a decision variable. This optimal control problem is suitable to problems whose solution points have indefinite switching frequency. Inspired by the method in [18], we replace the discrete switching sequence with some conventional control functions with interval range. For mixed state-control constraints, several relationships are then derived to link the original problem with the embedded problem by exploiting the method proposed by [18]. Unlike the Bengea's work, our main contributions can be shown in the following points:

(i) The applicable scope of the problem in [18] is expanded to handle a problem with mixed state-control constraints and multiple mode-switchings, and a numerical approximation algorithm is offered.

(ii) The algorithm we proposed is suitable to perform in a parallel machine because it meets the characteristics of parallelism. In this paper, a numerical method is developed in which switched optimal control problem can be decomposed into multiple switched modes, each which can be independently solved with proper time range. Finally, an active mode can be determined by comparing their largest values of the Hamiltonian functions associated with each switched mode.