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Stability of perturbed switched nonlinear systems with delays

Xingwen Liu [a,*], Yongbin Yu [b], Hao Chen [a]

[a] College of Electrical and Information Engineering, Southwest University for Nationalities of China, Chengdu, Sichuan, 610041, China
[b] School of Information and Software Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, China

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ABSTRACT

This paper addresses the stability problems of perturbed switched nonlinear systems with time-varying delays. It is assumed that the nominal switched nonlinear system (perturbation-free system) is uniformly exponentially stable and that the perturbations satisfy a linear growth bound condition. It is revealed that there exists an upper bound of perturbation guaranteeing that the perturbed system preserves the stability property of the nominal system, locally or globally, depending on both perturbations and the nominal system itself. An example is provided to illustrate the proposed theoretical results.

1. Introduction

Switched systems inherit the feature of both continuous state and discrete state dynamic systems and may possess very complicated dynamics due to switching between different subsystems [1,2]. It is noticed that remarkable achievements were made during the past several decades in the area of switched systems [3–5]. Because nonlinearities inevitably appear in various systems in the real world and have complicated impacts on system's performance [6–8], different dynamic properties of switched nonlinear systems have been investigated for a long period [9–11]. For some basic concepts and recent developments in this field, see [12–14] for details.

Two factors, time delays and perturbations, are often considered in research of dynamical systems. Delays, especially time-varying delays, are frequently encountered in diverse engineering systems, and may lead to performance deterioration and system malfunction [15]. Perturbations may result from modeling errors or aging and appear in real world engineering inevitably [16]. There are several different perturbations and each of them has different influence on the dynamics [17]. For example, the perturbed systems may behave as the perturbation itself provided that the nominal system is exponentially stable and the perturbation asymptotically approaches zero [18,19]. In many other cases, perturbations satisfy the so-called linear growth bound condition [20], which is considered in the present study. For perturbed nonlinear switched systems with time-varying delays, the bounded-input bounded-output stability was studied by means of a classical Lyapunov–Krasovskii method [21], and links between different stabilities of a class of switched nonlinear systems are revealed in [22].

Since a nominal system is generally easier to be modeled and analyzed than a perturbed system, it would be of great importance to infer the property of a perturbed system when the stability property of the nominal system is known. In [23, Lemma 9.1], it was proved that, for a delay-free nonlinear system which is exponentially stable, the perturbed system is

* Corresponding author.
E-mail address: xingwenliu@gmail.com (X. Liu).

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also exponentially stable provided that the perturbation satisfies a linear growth bound with a sufficiently small coefficient. Clearly, it is important to extend this result to nonlinear systems with delays. Therefore, this paper manipulates a more general case where the considered system is a switched nonlinear system with delays and perturbations, and imposes some conditions on perturbations so that the perturbed system can preserve the original system's stability property.

A number of relevant research papers have been reported. It was shown in [18] that, for a switched linear system with bounded or convergent perturbations, the perturbed system behavior is similar to the perturbation, provided that the nominal system is exponentially stable. Recently, the stability issue of discrete-time switched linear systems with time-varying delays and perturbations was investigated [24], which concluded that the perturbed system is exponentially stable if the nominal system is exponentially stable and the perturbation is small enough. Compared with the references mentioned above, the system involved in the present paper is with time-varying delays and perturbations and is a general switched nonlinear system. The task here is to explore whether or not a nonzero upper bound of perturbation exists to guarantee the perturbed system preserving the exponential stability of the nominal system.

Technically, handling such a problem is not trivial. In [23, Lemma 9.1], the Lyapunov function method was used, which is based on the fact that a nonlinear delay-free system is exponentially stable if and only if there exists a Lyapunov function; however, a similar conclusion is unavailable even in the context of switched linear systems with delays. In other words, the converse Lyapunov theorem does not apply here. Intuitively, if we just consider evolution of the perturbed system of a nominal system on a finite interval, one may claim the existence of the upper bound of perturbation with which the perturbed system and the nominal system have similar trajectories. However, as the considered interval approaches infinity, the upper bound may approach zero so that these two systems have similar stability property. The idea used here can be briefly descried as follows: First prove the existence of the maximum linear growth bound L of perturbation on a finite interval for which the trajectory of perturbed system can be bounded by a function, then, by means of mathematical inductive principle, prove that with the same L, the trajectory of perturbed system is bounded by the same function with exponentially decaying coefficient on the whole half right interval.

It is well-known that switching signal is an important factor affecting the dynamics of switched systems. For example, given subsystems of switched systems, different signals may result in quite different stability properties [25]. Furthermore, delays are required merely to be piecewise continuous and bounded in this paper, which is a very mild constraint. Indeed, in many reported papers, delays are required to be constant, continuously differentiable or slowly varying [26,27]. Therefore, we try to consider several commonly used switching signals and general delays so that the obtained results can be applied more widely.

The main contribution of the paper lies in the following two aspects: (1). With the assumption that perturbation satisfies a linear growth bound, two conditions are proposed which claim the existence of the maximum of the linear growth bound guaranteeing that the perturbed system may preserve the exponential stability of the nominal system, locally or globally, depending on the perturbation and the nominal system itself. (2). In the case of the perturbation being partially known, a tuning factor is introduced such that the "tuned" system can preserve the exponential stability of the nominal system.

The rest of this paper is organized as follows. Preliminaries and problems are presented in Section 2, main results are proposed in Section 3, and a numerical example is provided in Section 4. Finally, Section 5 concludes this paper.

Notation: A^{T} and A^{-1} are the transpose and inverse of matrix A, respectively. diag $(a_1, \ldots, a_n)$ is a diagonal matrix with diagonal elements $a_1, \ldots, a_n$. $\mathbb{R}$ ($\mathbb{R}_+$) is the set of real (positive) numbers, and $\mathbb{R}^n$ the n-dimensional real vector space. $\mathbb{R}^{n\times m}$ denotes the set of all real matrices of $n \times m$-dimension. $\mathbb{N}_0$ denotes the set of nonnegative integers and $\mathbb{N} = \mathbb{N}_0 \setminus \{0\}$. For any $m \in \mathbb{N}$, $\underline{m} = \{1, \ldots, m\}$ and $\underline{m}_0 = \underline{m} \cup \{0\}$. $\mathbb{R}_t = [t, \infty)$. $|a|$ is the absolute value of a real number a. The symbol $\mathbf{0}$ is an n-dimensional zero vector. For vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, $\boldsymbol{x} \succ (\succeq, \prec, \preceq)\boldsymbol{y}$ means that $\boldsymbol{x}$ is entrywise greater than (greater than or equal to, less than, less than or equal to) $\boldsymbol{y}$. These symbols can be applied to matrix in an obvious manner. $\mathcal{C}([a, b], X)$ is the set of continuous functions from interval $[a, b]$ to X. For any continuous function $\boldsymbol{x}(s)$ on $[-d, a)$ with scalars $a > 0, d > 0$ and any $t \in [0, a)$, $\boldsymbol{x}_t$ denotes a continuous function on $[t - d, t]$ defined by $\boldsymbol{x}_t(\theta) = \boldsymbol{x}(t + \theta)$ for each $\theta \in [-d, 0]$. Clearly, $\boldsymbol{x}_t(0) = \boldsymbol{x}(t)$. For any real number c, $c\boldsymbol{x}_t = c\boldsymbol{x}(t + \theta)$ for each $\theta \in [-d, 0]$; $\|\boldsymbol{x}_t\| = \sup_{t-d\le s\le t}\{\|\boldsymbol{x}(s)\|\}$. $\mathcal{C}_r([t - d, t], \mathbb{R}^n) = \{\boldsymbol{x} \in \mathcal{C}([t - d, t], \mathbb{R}^n) : \|\boldsymbol{x}\| \le r\}$. $\overline{\mathcal{B}}_r = \{\boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x}\| \le r\}$. Throughout this paper, the dimensions of matrices and vectors will not be explicitly mentioned if clear from context.

2. Problem statements and preliminaries

Consider the following switched system:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}_{\sigma(t)}(t, \boldsymbol{x}_t), \quad t \ge t_0 \\ \boldsymbol{x}(t) &= \boldsymbol{\phi}(t), \quad t \in [t_0 - d, t_0] \end{aligned} \tag{2.1}$$

where $t_0 \ge 0$, $\boldsymbol{x}(t) \in \mathbb{R}^n$ is state, the $\sigma : \mathbb{R}_{t_0} \to \underline{m}$ is a switching signal with m being the number of subsystems. It is always assumed that σ is with switching sequence $\{t_i\}_{i=0}^{\infty}$ satisfying $t_i > t_{i-1} (\forall i \in \mathbb{N})$ and $\lim_{i\to\infty} t_i = \infty$ and that σ is piecewise constant and continuous from the right, that is, for any $i \in \mathbb{N}$, there exists $l \in \underline{m}$ such that $\sigma(t) = l$, $t \in [t_{i-1}, t_i)$. $\boldsymbol{\phi} \in \mathcal{C}([t_0 - d, t_0], \mathbb{R}^n)$ is an initial vector-valued function. For each $l \in \underline{m}$, $\boldsymbol{f}_l$ maps $\mathbb{R}_{t_0} \times \mathcal{C}([t - d_{2l}, t - d_{1l}], \mathbb{R}^n)$ into $\mathbb{R}^n$ with d_{1l} and d_{2l} being constants, $0 \le d_{1l} \le d_{2l}$, $d = \max_{l\in\underline{m}}\{d_{2l}\}$. The following assumption is always imposed on system (2.1) when local stability is considered: