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First steps toward formal controller synthesis for bipedal robots with experimental implementation

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ABSTRACT

Bipedal robots are prime examples of complex cyber–physical systems (CPSs). They exhibit many of the features that make the design and verification of CPS so difficult: hybrid dynamics, large continuous dynamics in each mode (e.g., 10 or more state variables), and non-trivial specifications involving nonlinear constraints on the state variables. In this paper, we propose a two-step approach to formally synthesize controllers for bipedal robots so as to enforce specifications by design and thereby generate physically realizable stable walking. In the first step, we design outputs and classical controllers driving these outputs to zero. The resulting controlled system evolves on a lower dimensional manifold and is described by the hybrid zero dynamics governing the remaining degrees of freedom. In the second step, we construct an abstraction of the hybrid zero dynamics that is used to synthesize a controller enforcing the desired specifications to be satisfied on the full order model. Our two step approach is a systematic way to mitigate the curse of dimensionality that hampers the applicability of formal synthesis techniques to complex CPS. Our results are illustrated with simulations showing how the synthesized controller enforces all the desired specifications and offers improved performance with respect to a classical controller. The practical relevance of the results is illustrated experimentally on the bipedal robot AMBER 3.

1. Introduction

Legged robots are complex dynamic cyber–physical systems (CPSs). As a concrete example, consider MABEL shown in Fig. 1. This bipedal robot possesses nonlinear, non-order preserving, non-convex dynamics described by a hybrid model with 14 state variables and four actuators [1]. To enable MABEL to accept a set of high-level locomotion commands over a network, and successfully execute the commands while responding automatically and safely to uncertainty in the assumed

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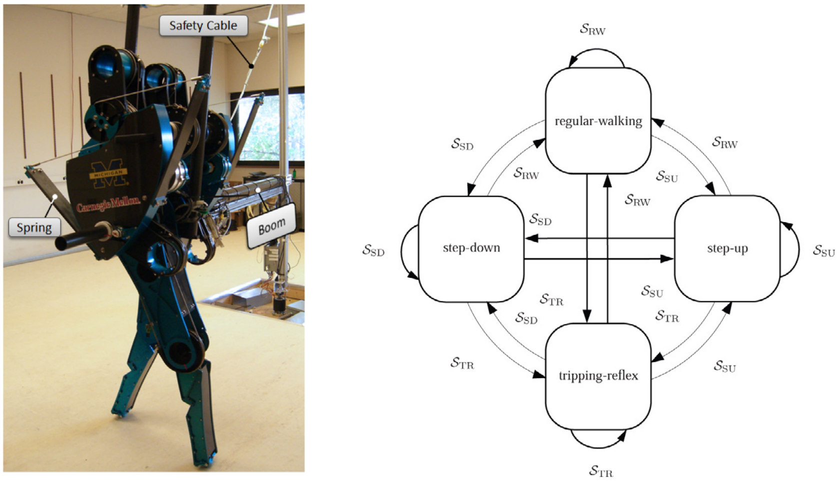

Fig. 1. The planar (2D) biped MABEL was developed for the study of dynamic locomotion. High-level motion primitives were used on MABEL to allow walking over rough ground without tripping. While tools for automatic low-level control algorithm synthesis are well developed, at the state machine level, all tuning was done by hand for lack of appropriate tools. "Probable correctness" was established through extensive simulations and experiments.

profile of the environment, the finite state machine shown in Fig. 1 was designed [2]. It allowed MABEL to compose on the fly a set of low-level control algorithms executing a handful of motion primitives. A team of graduate students hand-tuned the transition conditions among the various nodes of the state machine. Each time a small change was made in one of the software or hardware components, such as adjusting a transition condition or adding a sensor, the entire state machine had to be completely retested, leading often to the redesign of other software components. *There is a pressing need to understand this, and more general CPSs, in a way that allows for the automatic synthesis of embedded control software that is provably correct by construction.*

In this paper, we begin to lay the groundwork for this correct-by-construction control software design process in the context of dynamic systems. In particular, the specific bipedal robot that will be studied is AMBER 3 as shown in Fig. 2. We consider a walking gait with the simplest discrete structure, resulting in a single-mode hybrid model with 12 state variables and 6 actuators. While we seek formal guarantees on the behavior of the 12-dimensional closed-loop system, we do not propose to perform formal synthesis on a model this large. Similarly to the work in [3,4], we focus on the regulation of a subset of system states and use advanced nonlinear control methods to transform the complex dynamics to a simpler, more tractable system which is amenable to the correct-by-construction synthesis techniques. In contrast to [3,4], where the authors exploit differential flatness to reduce the nonlinear synthesis problem to a controller design problem for a chain of integrators, our method applies to the aforementioned hybrid system with non-flat outputs. Specifically, in our approach the chain of integrators is forced to be at equilibrium and we apply the symbolic abstraction techniques to a hybrid system that lives on an attractive, hybrid-invariant, low-dimensional manifold which is "complementary" to the state space of the integrators [5,6]. The low-dimensional hybrid subsystem is called the Hybrid Zero Dynamics (HZD), and its solutions can be used to reconstruct the solutions of the high-dimensional hybrid system. The end result is the ability to guarantee specifications on the full-order high-dimensional system via the reduced order representation encoded by the HZD.

There is a growing interest in the synthesis of correct-by-construction controllers for robotic applications as evidenced by the growing body of work on this topic [7–10]. Although the techniques we employ for synthesis are based on the symbolic abstraction techniques described in [11], what sets the results in this paper apart from prior work is the complexity of the system being controlled. In particular, as previously mentioned, the hybrid model for AMBER 3 requires 12 state variables, which is larger than any system previously reported in the literature for which correct-by-construction control has been synthesized. The key to scaling the symbolic controller synthesis techniques to this level of complexity is the new design flow based on the HZD. This is the main contribution of the paper as we believe that its applicability transcends the specific formal synthesis technique we employ and the robotics domain in which we develop the result.

The results in this paper are based on previous work by the authors on two lines of research that have been independently pursued in the past: (1) control of bipedal robots via hybrid zero dynamics and (2) synthesis of controllers via finite-state abstractions. In order to combine techniques from these two different areas several new results, formalized in Theorems 1 through 4, had to be proved. Theorems 1 and 2 are new because they represent a notion of physically realizable walking that had not been treated previously in the context of hybrid zero dynamics; specifically, previous work focused on asymptotic stability of periodic trajectories lying in the zero dynamics manifold, while for the present work, a more general notion of aperiodic upright walking gaits is required. Theorem 3 is new since prior work by the authors on the construction of abstractions for hybrid systems considered only switched systems. The hybrid model considered in this paper, as presented in Section 2, is not a switched system since it is equipped with a nontrivial guard and reset map

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