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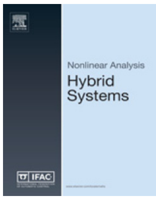

Parameter to state stability of control Lyapunov functions for hybrid system models of robots[☆]

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ABSTRACT

Model based controllers, by virtue of their dependence on a specific model, are highly sensitive to imperfections in model parameter estimation leading to undesirable behaviors, especially in robots that undergo impacts. With the goal of quantifying the effect of model imperfection on the resulting output behavior from a control Lyapunov function (CLF) based controller, we formally derive a measure for model parameter mismatch and show that a bounded measure leads to an ultimate bound on the CLF. This is also extended to the discrete map by introducing an impact measure. The measure is controller and path dependent, and not just parameter dependent, thereby differentiating it from existing methods. More specifically, if traditional methods yield ultimate boundedness for a bounded parameter uncertainty, the proposed "measure" uses the notion of input to state stability (ISS) criterion to establish stability of model based controllers. The main result of this paper establishes that the proposed CLF based controller is parameter to state stable (PSS) for a class of robotic hybrid systems—systems with impulsive effects. These formal results motivate the construction of a robust controller – combining a computed torque term with a traditional PD term – that yields stricter convergence rates and bounds on the errors. This is demonstrated on the bipedal robot AMBER with a modeling error 30%, wherein the stability of the proposed controller is verified in simulation.

1. Introduction

There are two main paths for approaching the problem of model parameter uncertainty in mechanical systems: 1. Obtain (usually through exhaustive experimentation) an accurate identification of the model and then adopt a stabilizing controller. 2. Develop a robust controller that renders the system stable despite the uncertainty. For the first approach, many methods have been explored in identifying the model parameters involving state estimation, regression, determination and validation in a systematic manner [1,2]; this often involves substantial and time consuming experimental validation [3]. By determining an accurate model, model dependent controllers can be applied to realize accurate tracking and control of such systems. Despite its simplistic nature, the success of this tedious approach typically relies on the accuracy of the estimation while accounting for variations of parameters over time. While these model dependent controllers are able to deliver on the

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[☆] A preliminary version of this manuscript was published in the IFAC conference on ADHS 2015 (Kolathaya and Ames, (2015)). The present version shows the input to state stability (ISS) property, i.e., zero stability for zero uncertainty, and boundedness for a bounded measure of uncertainty. Rephrasing, the current version shows parameter to state stability of periodic orbits for hybrid robotic systems. The paper also elaborates the proofs of Theorem 1 and 2 in Kolathaya and Ames (2015) including the detailed procedure for computing the ultimate bounds for the CLF based controller: computed torque+PD.

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performance (exponential convergence, large domains of attraction) promised by the formal controller design process, they are extremely sensitive to changes in the parameters sometimes leading to instability.

There is a significant amount of work in literature that take the second approach, i.e., relax the need for an accurate model [4–7]. Some of the methods even completely eliminate the requirement of the information of the entire parameter set via adaptive control [8,9], and via PD and PID regulation and tracking [10–16]. [17,18] achieved adaptive control in bipedal robots without considering the impact models. L1 adaptive control was implemented in [19,20] to yield an ultimate bound on the tracking errors. There is also a significant amount of work done on developing controllers that yield a bounded output error for a bounded parameter uncertainty [4,5]. While all these methods lead to the development of a robust controller that renders the system stable for a bounded uncertainty, the tracking and regulation performance is sacrificed, which is critical in systems that undergo rapid changes in states—hybrid systems with impulsive effects.

It is important to note that the concept of a bounded error output for a bounded parameter uncertainty has proven to be extremely restrictive on the choice of available controllers. For example, it is a well known fact that the swinging motion of a simple pendulum with zero input (trivial input) is independent of the point mass at its end. This simple example demonstrates that we can always realize a space of unbounded model parameter sets that have the exact same response for the given control input. This motivates the need for a formal framework to understand the relationship between model parameter uncertainty and the resulting tracking/regulation performance—especially in the context of hybrid system models of robotic systems. In order to properly quantify this uncertainty that can be formally related to the resulting stability of the system, a *measure* was defined in [21] and verified in simulation in the robot AMBER. Therefore, the goal of this paper is to expand on [21] and establish and prove stability properties by using the notion of input to state stability (ISS) in a formal manner.

For systems of the form $\dot{x} = f(\Theta; x, u)$, where Θ represents the parameter set, x represents the state and u the control input, the class of controllers that achieve a desired control objective, e.g., driving $x \to 0$, can be written via the control Lyapunov function (CLF) $V(x) > 0$, through the set of control inputs that satisfy the derivative condition that V decreases along solutions:

$$K = \{u \in \mathrm{U} : L_f V(x, u) \leq 0\}. \tag{1}$$

Therefore model dependent controllers, like feedback linearization [22] and adaptive control [23] can be reformulated via CLFs which satisfy the condition: $V \to 0 \implies x \to 0$. Since $\dot{V}(x, u) = \frac{\partial V}{\partial x} f(\Theta; x, u)$ is a function of the vector field f, determination of u depends consequently on the parameters Θ. But, if the controller (say CLF) that stabilizes the known model is applied on the imperfect model, the resulting dynamics of this imperfect model satisfies the conditions of an Input to State Stable (ISS)-Lyapunov function [24]. The ISS-Lyapunov function is constructed w.r.t. the input that is a function of the uncertainty. Furthermore, for robotic systems, this function can be written as a linear function of the error in parameters Θ. Therefore, by defining a measure that quantifies the parameter uncertainty as a function of the path and the controller, we can construct robust controllers that yield strict ultimate bounds for the specified uncertainty in the model. Further, as an improvement on the performance, we can construct controllers that use a combination of model based and non model based controllers (computed torque+PD) to obtain exponential ultimate bounds for hybrid systems.

The primary goal of this paper is to show that, by using the notion of input to state stability (ISS), CLFs with a model mismatch can be shown to be Parameter to State Stable for the nonlinear hybrid system model of a robotic system undergoing impacts. In other words, despite the differences in the model, the model based CLF based controller will still yield a bounded output error for a bounded function of parameter uncertainty. To establish this fact, a measure for parameter uncertainty for both the continuous and discrete map will be defined. This will be illustrated through the consideration of a representative robotic system: the bipedal robot AMBER (shown in Fig. 1). There are other approaches like [25] that use control Lyapunov functions to achieve exponential convergence to zero under bounded uncertainty. Our objective is the same, i.e., utilize control Lyapunov functions to obtain exponential convergence, but, to an ultimate bound (in other words, convergence to small acceptable tracking errors). This formal construction helps in obtaining controllers that are not only highly convergent, but also robust to the model mismatch, getting the best from the two worlds. It will be shown with a CLF based controller, computed torque+PD, resulting in a stable walking gait for the robot in simulation.

The paper is structured in the following fashion: Section 2 introduces the notion of input to state stability (ISS) for both continuous and discrete time systems. The framework to show *parameter uncertainty to state stability* will be built on this notion. Section 3 introduces the robot model and the control methodology used—control Lyapunov functions obtained through the method of computed torque. Section 4 studies this controller in the context of an uncertain model of the robot and characterizes the resulting uncertain behavior through Lyapunov functions. In Section 5, the resulting uncertain dynamics exhibited by the robot is measured formally through the construction of a measure that quantifies parameter uncertainty, which is the main formulation of this paper on which the formal results will build. It will be shown that there is a stronger relationship between the error bounds and the *parameter measure* than the bounded input bounded output estimate, which motivates the introduction of a robust auxiliary controller: computed torque+PD. This will be utilized for establishing bounds for the entire dynamics, under the assumption of a stable limit cycle in the zero dynamics. This method is extended to hybrid systems through the introduction of an impact measure in Section 6. Under the assumption that the hybrid zero dynamics contains a stable periodic orbit, the computed torque controller appended with the auxiliary input is applied on the model, which results in bounded dynamics of the underactuated hybrid system. The paper concludes with

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