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Hybrid optimal control of an electric vehicle with a dual-planetary transmission

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ABSTRACT

A hybrid systems framework is presented for the analysis and optimal control of an electric vehicle equipped with a seamless dual stage planetary transmission. A feature of special interest is that, due to the perpetual connectedness of the motor to the wheels via the seamless transmission, the mechanical degree of freedom changes during the transition period. These circumstances where autonomous and controlled state jumps at the switching instants are accompanied by changes in the dimension of the state space are reflected in the definition of hybrid systems and the corresponding statement of the Hybrid Minimum Principle (HMP). Furthermore, the state-dependent motor torque constraints which impose mixed input-state constraints are converted to state-independent input constraints via a change of variables and the introduction of auxiliary discrete states. Optimal control problems for the minimization of acceleration duration and the minimization of energy consumption for the acceleration task are formulated within the presented framework and simulation results are presented for the optimal control inputs and the optimal gear changing instants for reaching the speed of 100 km/hr from the stationary initial condition. A phenomenon of note that appears in the dynamical evolution of the vehicle is the presence of power regeneration as a part of the acceleration task for the minimization of the energy consumption.

1. Introduction

The Minimum Principle (MP), also called the Maximum Principle in the pioneering work of Pontryagin et al. [1], is a milestone of systems and control theory that led to the emergence of optimal control as a distinct field of research. This principle states that any optimal control along with the optimal state trajectory must solve a two-point boundary value problem in the form of an extended Hamiltonian canonical system, as well as an extremization condition of the Hamiltonian function. Whether the extreme value is maximum or minimum depends on the sign convention used for the Hamiltonian definition. The generalization of the Minimum Principle for hybrid systems, i.e. control systems with both continuous and discrete states and dynamics, results in the Hybrid Minimum Principle (HMP) (see e.g. [2–14]). The HMP gives necessary conditions for the optimality of the trajectory and the control inputs of a given hybrid system with fixed initial conditions and a sequence of autonomous and controlled switchings. These conditions are expressed in terms of the minimization of the distinct Hamiltonians indexed by the discrete state sequence of the hybrid trajectory. A feature of special interest is the boundary conditions on the adjoint processes and the Hamiltonian functions at autonomous and controlled

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switching times and states; these boundary conditions may be viewed as a generalization of the optimal control case of the Weierstrass–Erdmann conditions of the calculus of variations [15].

The goal of this paper is to present a hybrid systems formulation of an electric vehicle equipped with a dual-stage planetary transmission presented in [16,17] and employ hybrid optimal control theory to find the optimal inputs for the gear changing problem for electric vehicles. The seamless dual brake transmission under study is designed particularly for electric vehicle in order to reduce the size of the electric motor and provide an appropriate balance between efficiency and dynamic performance (see [16–18] and the references therein for more discussion about industrial motivations for the particular design). Due to the special structure of the transmission under study, the mechanical degree of freedom and therefore, the dimension of the (continuous) state space of the system depend on the status of the transmission, i.e. whether a gear number is fixed or the system is undergoing a transition between the two gears. Therefore, the modelling of the powertrain requires the consideration of autonomous and controlled state jumps accompanied by changes in the dimension of the state space.

These characteristics are reflected in the definition of hybrid systems in Section 2 and Appendix A, where a general class of nonlinear systems on Euclidean spaces are considered whose dimensions depend upon their corresponding discrete states, and where autonomous and controlled switchings and jumps between different state spaces are allowed at the switching states and times. A general class of optimal control problems associated with the presented hybrid systems framework is introduced with a large range of running, terminal and switching costs. Other hybrid optimal control frameworks presented in the rich literature on hybrid optimal control theory (see e.g. [2–10,19–26]) may be obtained via variations and specializations of the framework presented here. In Section 3, the statement of the Hybrid Minimum Principle for the class of hybrid optimal control problems under study is presented. Distinctive aspects of the HMP presented here in comparison with other versions of the HMP are the presence of state dependent switching costs, the possibility of state space dimension change, and the existence of low dimensional switching manifolds.

In Section 4 we extend the formulation presented in [18] for the dynamics and energy consumption of gear-equipped electric vehicles by the inclusion of the transmission dynamics, considering the model of a seamless dual brake transmission reported in [16,17]. After presenting the Kinematic relations in the driveline, the dynamics of the powertrain is derived from the Principle of Virtual Work and the generalized Euler–Lagrange equation. In order to avoid state-dependent input constraints imposed by the maximum torque and maximum power constraints of the electric motor (see also Fig. 2) the state-dependent input constraints are converted to state-independent constraints via a change of variables and the introduction of auxiliary discrete states. The corresponding hybrid systems formulation which is an extension of [27] is presented in Section 5.

In Section 6 the hybrid optimal control problem is considered to be the minimization of the time required for reaching the speed of 100 km/h from the stationary state (see also [27]). In Section 7 a similar manoeuvre is considered but, in contrast, the minimization of the energy consumption is studied for performing the same task on a longer time horizon. The Hybrid Minimum Principle is employed to find the optimal control inputs and optimal switching instants for both problems and simulation results are presented. A phenomenon of note that appears in the dynamical evolution of the vehicle in the energy optimal mode is the presence of power regeneration as a part of the optimal control for the acceleration task.

2. Hybrid optimal control problems

Informally speaking, a hybrid system is a control system whose state is composed of both discrete state components $q \in Q$, and continuous state components $x \in \mathbb{R}^{n_q}$, and whose input is composed of both discrete input components $\sigma \in \Sigma$, and continuous input components $u \in U_q$, for which the evolution of the continuous state [component] is governed by a set of controlled vector fields $f_q \in F$, in the form of

$$\dot{x}_{q_i}(t) = f_{q_i}\left(x_{q_i}(t), u_{q_i}(t)\right), \quad \text{a.e. } t \in [t_i, t_{i+1}), \tag{1}$$

subject to initial and boundary conditions

$$x_{q_0}(t_0) = x_0, \tag{2}$$

$$x_{q_j}(t_j) = \xi_{\sigma_{q_{j-1}q_j}}\left(x_{q_{j-1}}(t_j-)\right) \equiv \xi_{\sigma_{q_{j-1}q_j}}\left(\lim_{t\uparrow t_j} x_{q_{j-1}}(t)\right), \tag{3}$$

where $0 \le i \le L$, $1 \le j \le L$, $t_{L+1} = t_f < \infty$, and at the switching instants t_j, the updates in the discrete state [component] are governed by a finite automata A and its corresponding transition map Γ. Switchings are referred to as autonomous switchings if they are constrained upon transversal arrival of the continuous state trajectory on a switching manifold described locally by

$$m_{q,r} = \left\{x \in \mathbb{R}^{n_q} : m^1_{q,r}(x) = 0 \wedge \cdots \wedge m^k_{q,r}(x) = 0\right\}, \tag{4}$$

where $m_{q,r} \in \mathcal{M}$, $q, r \in Q$, $r \in A(q,\sigma)$, $\sigma \in \Sigma$, and $1 \le k \le n_q$. Controlled switchings, in contrast, are direct results of the (hybrid) input command.

The overall hybrid dynamics can be described in a hybrid automata diagram as illustrated in Fig. 3 for an electric vehicle equipped with a dual planetary transmission studied in this paper.