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Robust global recurrence for a class of stochastic hybrid systems

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ABSTRACT

We study a weak property called recurrence for a class of stochastic hybrid systems and establish robustness of the recurrence property. In particular, we establish that recurrence of an open, bounded set is robust to sufficiently small perturbations in the set, perturbations of the data of the stochastic hybrid system and modifications to the system data that slow down the recurrence property. The robustness results are a consequence of the mild regularity properties assumed for the stochastic hybrid system.

1. Introduction

Stochastic hybrid systems (SHS) combine continuous-time evolution, discrete-time events and probabilistic behavior. Frameworks for modeling SHS are in [1–4]. SHS models arise frequently in the context of complex systems like air traffic management systems, networked control systems and systems biology. See [5–7] for more details. The recent survey paper [8] presents a unified modeling framework for the various SHS representations in the literature and addresses stability related issues. In particular, important topics that are well studied in the case of non-stochastic hybrid systems, like sufficient conditions for stability, weak sufficient conditions for stability, invariance principle, robust stability conditions and converse Lyapunov theorems, are analyzed in detail in [8] for stochastic hybrid systems.

In this paper, we address one aspect of the future research directions proposed in [8]. We study a weak stochastic property called recurrence and establish robustness of recurrence to various state dependent perturbations. The class of systems for which we study this property is stochastic hybrid systems with randomness restricted to the discrete-time dynamics. The system model we study can account for spontaneous transitions, forced transitions and probabilistic resets. While there are many interesting developments in the area of stochastic differential inclusions, the results needed to establish robustness particularly sequential compactness of solutions with mild regularity assumptions on the system data are not available yet and hence a more general study of such systems is a challenge.

We adopt the framework for modeling SHS with non-unique solutions proposed in [2,9]. This class of systems covers other frameworks such as piecewise-deterministic Markov processes (PDMP) and Markov jump systems. We study stochastic systems with non-unique solutions for two key reasons. Firstly, analyzing such systems is crucial to developing a robust stability theory. Robust stability for non-stochastic hybrid systems is studied in [10]. Robustness of the stability property is also crucial in developing converse Lyapunov theorems (see [11,12]). Secondly, such system models allow flexibility in control design applications (see [13–15]).

The stochastic property of interest in this paper is called recurrence. Loosely speaking, recurrence of an open set implies that solutions visit the set infinitely often with probability one. The recurrence property is frequently studied in the literature

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on stochastic systems. See [1,16]. Recurrence is a weaker notion of stability compared to more commonly studied notions of mean square asymptotic stability and asymptotic stability in probability. Generally, recurrence of a set need not imply any stability-like property or invariance-like property for the set. In fact, recurrence of the set does not necessarily imply solutions stay bounded in a probabilistic sense. Still, recurrence is useful to study in situations where stronger properties like convergence or asymptotic stability cannot be established. See [17] for an example in the context of source-seeking by a MAV (Micro Aerial Vehicle) where persistent disturbances prevent establishing stronger properties. Recurrence can also be used in situations to provide a sharper characterization for the behavior of solutions even though asymptotic stability properties may be satisfied by certain sets. See [12, Example 1] for more details.

The recurrence property when studied for non-stochastic systems with respect to bounded sets is equivalent to the well studied property of ultimate boundedness. The recurrence property is studied with respect to open sets because for closed sets recurrence is generally not equivalent to uniform recurrence and need not be robust. See [12] for more details. The equivalence between recurrence of bounded sets and ultimate boundedness of solutions does not hold in general for stochastic systems.

Robustness of stability can be loosely defined as the stability property for the nominal system being preserved when the system is affected by sufficiently small perturbations. In [18] results on robustness of recurrence for a class of stochastic hybrid systems are explored. The robustness results in [18] exploit the regularity properties of the nominal system. In this work, we provide detailed proofs and examples highlighting the main results established in [18]. The results proved in this paper can be used to establish a converse Lyapunov theorem for recurrence in stochastic hybrid systems.

The rest of the paper is organized as follows. Section 2 presents the basic notation and definitions to be used in the paper. Section 3 introduces the SHS framework that will be considered in the rest of the paper. The recurrence property is explained in Section 4. Section 5 introduces viability and reachability probabilities which will be used to prove the main results of the paper. Section 6 establishes some basic bounds related to these probabilities. The main results related to robustness of the recurrence property to various state dependent perturbations are presented in Section 7. The proof of the main results in the paper rely on the properties of viability and reachability probabilities introduced in [2] and the results on sequential compactness of solutions for a class of stochastic hybrid systems established in [19]. Section 8 includes a short discussion of the application of the robustness results in developing a converse Lyapunov theorem for recurrence. Section 9 presents some concluding comments. The Appendix contains proofs of the results from Sections 6–7.

2. Basic notation and definitions

For a closed set $S \subset \mathbb{R}^n$ and $x \in \mathbb{R}^n$, $|x|_S := \inf_{y\in S} |x - y|$ is the Euclidean distance of x to S. $\mathbb{B}$ (resp., $\mathbb{B}^o$) denotes the closed (resp., open) unit ball in $\mathbb{R}^n$. Given a closed set $S \subset \mathbb{R}^n$ and $\epsilon > 0$, $S + \epsilon\mathbb{B}$ (resp., $S + \epsilon\mathbb{B}^o$) represents the set $\{x \in \mathbb{R}^n : |x|_S \le \epsilon\}$ (resp., $\{x \in \mathbb{R}^n : |x|_S < \epsilon\}$). $\mathbb{R}_{\ge 0}$ denotes the non-negative real numbers; $\mathbb{Z}_{\ge 0}$ denotes the non-negative integers. Let $\mathcal{T}$ be a topological space. A function $\Psi : \mathcal{T} \to \mathbb{R}_{\ge 0}$ is *upper semicontinuous* if for every converging sequence $\{t_i\} \to t$, $\limsup_{i\to\infty} \Psi(t_i) \le \Psi(t)$. For $S \subset \mathbb{R}^n$, the symbol $\mathbb{I}_S$ denotes the indicator function of S i.e., $\mathbb{I}_S(x) = 1$ for $x \in S$ and $\mathbb{I}_S(x) = 0$ otherwise. Following [2], we define for sets $S_1, S_2 \subset \mathbb{R}^n$, $\mathbb{I}_{\subset S_1}(S_2) = 1 - \sup_{x\in S_2} \mathbb{I}_{\mathbb{R}^n\setminus S_1}(x)$ and $\mathbb{I}_{\cap S_1}(S_2) = \sup_{x\in S_2} \mathbb{I}_{S_1}(x)$ with the convention that the maximum's are zero when $S_2 = \emptyset$. For $\tau \ge 0$, we define the set $\Gamma_{\le\tau} := \{(s,t) \in \mathbb{R}^2 : s + t \le \tau\}$. The sets $\Gamma_{<\tau}$, $\Gamma_{\ge\tau}$ are defined similarly. A set-valued mapping $M : \mathbb{R}^p \rightrightarrows \mathbb{R}^n$ is *outer semicontinuous* if, for each $(x_i, y_i) \to (x, y) \in \mathbb{R}^p \times \mathbb{R}^n$ satisfying $y_i \in M(x_i)$ for all $i \in \mathbb{Z}_{\ge 0}$, $y \in M(x)$. A mapping M is *locally bounded* if, for each bounded set $K \subset \mathbb{R}^p$, $M(K) := \bigcup_{x\in K} M(x)$ is bounded. $\mathbf{B}(\mathbb{R}^m)$ denotes the Borel σ-field. A set $F \subset \mathbb{R}^m$ is measurable if $F \in \mathbf{B}(\mathbb{R}^m)$. For a measurable space $(\Omega, \mathcal{F})$, a mapping $M : \Omega \rightrightarrows \mathbb{R}^n$ is $\mathcal{F}$*-measurable* [20, Def. 14.1] if, for each open set $\mathcal{O} \subset \mathbb{R}^n$, the set $M^{-1}(\mathcal{O}) := \{\omega \in \Omega : M(\omega) \cap \mathcal{O} \ne \emptyset\} \in \mathcal{F}$. The functions $\pi_i : \mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0} \times \mathbb{R}^n \to \mathbb{R}_{\ge 0}$ are such that $\pi_i(t_1, t_2, z) = t_i$ for each $i \in \{1, 2\}$. A function $\kappa : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ belongs to class-$\mathcal{K}$ if it is continuous, strictly increasing and $\kappa(0) = 0$. A function $\kappa : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ belongs to class-$\mathcal{K}_\infty$ it if belongs to class-$\mathcal{K}$ and is unbounded.

3. Preliminaries on stochastic hybrid systems

We consider a class of SHS studied in [2]. Let the state $x \in \mathbb{R}^n$ and random input $v \in \mathbb{R}^m$. Then, the SHS is written formally as

$$\dot{x} \in F(x), \quad x \in C \tag{1a}$$

$$x^+ \in G(x, v^+), \quad x \in D \tag{1b}$$

$$v \sim \mu(\cdot) \tag{1c}$$

where $C, D \subset \mathbb{R}^n$ represent the flow and jump sets respectively and $F : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$, $G : \mathbb{R}^n \times \mathbb{R}^m \rightrightarrows \mathbb{R}^n$ represent the flow and jump maps respectively. So, the continuous-time dynamics are modeled by a differential inclusion and the discrete-time dynamics are modeled by a stochastic difference inclusion.

The distribution function μ is derived from the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a sequence of independent, identically distributed (i.i.d.) input random variables $\mathbf{v}_i : \Omega \to \mathbb{R}^m$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ for $i \in \mathbb{Z}_{\ge 1}$. Then μ is defined as $\mu(A) = \mathbb{P}(\omega \in \Omega : \mathbf{v}_i(\omega) \in A)$ for every $A \in \mathbf{B}(\mathbb{R}^m)$. We denote by $\mathcal{F}_i$ the collection of sets $\{\omega : (\mathbf{v}_1(\omega), \ldots, \mathbf{v}_i(\omega)) \in A\}$,

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