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Stability of switched nonlinear time-delay systems with stable and unstable subsystems

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ABSTRACT

This paper investigates the stability of switched nonlinear time-delay systems with stable and unstable subsystems. Several stability criteria are presented by resorting to novel inequality technique and average dwell time approach, which relax the assumptions that all subsystem matrices are commutative pairwise and Hurwitz stable. Finally, two numerical examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

Switched systems are a type of hybrid systems, which comprise a collection of subsystems equipped with a switching law orchestrating among these systems. The widespread applications of switched systems are also due to increasing performance requirements in controls, such as power systems [1], automated highways [2], multi-agent systems [3], and aircraft control systems [4]. Thus, switched systems have been attracting more attention in the control community, and many stability analyses and stabilization results related to switched systems have been reported in the literature [5–10]. To mention a few, in [11], Morse showed that when all modes are exponentially stable the entire switched system is exponentially stable under any switching signal if the time between two successive switchings, called the dwell time, is sufficiently large. Hespanha and Morse [12] extended the concept of "dwell time" to the concept of "average dwell time". Based on average dwell time method and multiple Lyapunov functions technique, Lin et al. [13] considered finite-time boundedness and L_2-gain analysis for switched delay systems with norm-bounded disturbance. In [14], Zhao et al. introduced the mode-dependent average dwell time method to study the stability and stabilization of switched linear systems.

However, the above-mentioned papers are mainly concerned with switched linear systems. Practically, most physical systems, chemical systems, biological systems, and man-made systems are nonlinear [15]. Compared with linear systems, theory of nonlinear systems is less developed due to their inherent complexities. It is essential to extend the control approaches of switched linear systems to switched nonlinear systems. As is known to us, a large number of nonlinear systems can be presented as the sum of a linear part and a nonlinear part, and many researchers have paid more attention to studying a class of such systems, e.g., [16–27] and the references therein. Zhai et al. [28] used the average dwell approach to investigate switched systems with nonlinear perturbations, while Alwan and Liu [21] studied the linear and weakly nonlinear switched delay systems with stable and unstable subsystems and obtained several exponential stability criteria

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by using multiple Lyapunov functions technique and dwell-time approach. Recently, Dong [29] presented stability criteria of switched systems with general nonlinear disturbances by employing the generalized Gronwall–Bellman inequality, and the results were further improved in [30]. Note that the subsystem matrices in [29,30] are required to be commutative pairwise and Hurwitz stable. Now the following question arises. Could we also establish the stability criteria of switched nonlinear time-delay systems with unstable modes? This motivates us for the study.

In this paper, we investigate the stability of switched nonlinear time-delay systems with both Hurwitz stable and unstable subsystems. The main contribution of this paper is mainly threefold: (1) different from the method in [30], several novel criteria are obtained by using average dwell time approach; (2) the subsystem matrices in this paper may be non-commutative and Hurwitz unstable; (3) compared with the systems studied in [28,21], the perturbations may have stronger nonlinearities which satisfy the superlinear or sublinear conditions.

The rest of this paper is organized as follows. In Section 2, a switched nonlinear time-delay system is introduced and some useful definition and lemma are also presented. We state and prove our main results in Section 3. In Section 4, two examples are provided to illustrate the main results. Finally, Section 5 gives a conclusion.

Notations: The notations are quite standard. Throughout the paper, let $R = (-\infty, +\infty)$, $R_+ = [0, +\infty)$, R^n be the n-dimensional Euclidean space, and $R^{n\times n}$ be the set of $n \times n$ real matrices. For $x \in R^n$, $\|x\|$ denotes the Euclidean norm of x. $P > 0$ means that P is a symmetric and positive definite matrix. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are the maximum and minimum eigenvalues of A, respectively, and $\|A\| = \sqrt{\lambda_{\max}(A^TA)}$.

2. Problem description and preliminaries

Consider a class of switched nonlinear time-delay systems

$$\begin{cases} x'(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-\tau(t)) + f_{\sigma(t)}(t, x(t), x(t-\tau(t))), & t \ge t_0, \\ x(t) = \varphi(t), & t \in [t_0 - \tau_2, t_0], \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $\sigma(t) : [0, +\infty) \to \Lambda = \{1, 2, \ldots, L\}$ is a piecewise constant function of time, called a switching signal, L is the number of subsystems. For a switching sequence $0 \le t_0 < t_1 < \cdots < t_j < t_{j+1} < \cdots$, $\sigma(t)$ is everywhere continuous from the right, when $t \in [t_j, t_{j+1})$, we say the $\sigma(t_j)$th subsystem is active. $\tau(t)$ is a time-varying delay satisfying $0 \le \tau_1 \le \tau(t) \le \tau_2$, where τ_1 and τ_2 are constant scalars. $\varphi \in C([t_0 - \tau_2, t_0], R^n)$ is the initial function of the system, where $C([t_0 - \tau_2, t_0], R^n)$ is the Banach space of all continuous functions $\varphi(t) : [t_0 - \tau_2, t_0] \to R^n$ with $\|\varphi\|_{\tau_2} = \sup_{t\in[t_0-\tau_2,t_0]} \|\varphi(t)\|$. A_i and B_i are known real constant matrices with appropriate dimensions. For any $i \in \Lambda$, $f_i(t, x, y)$ is a continuous function which satisfies

$$\|f_i(t, x, y)\| \le \varepsilon_1\|x\| + \varepsilon_2\|y\| + \varepsilon_3\|x\|^p + \varepsilon_4\|y\|^p + \gamma(t), \quad i \in \Lambda, \tag{2.2}$$

where p is a constant satisfying $p > 0$ and $p \ne 1$, $\varepsilon_i \ge 0$ $(i = 1, 2, 3, 4)$ are constants, and $\gamma(t)$ is a nonnegative Lebesgue integrable function such that $\int_{t_0}^{+\infty} e^{\lambda(t-t_0)}\gamma(t)dt < +\infty$, where λ will be specified later. The solutions of systems (2.1) are those which are absolutely continuous functions defined on interval $[t_0 - \tau_2, +\infty)$.

Remark 1. Different from most of the existing results where the nonlinear parts are bounded by a linear function, the assumptions about the nonlinear parts given in (2.2) are more general, which allows the system contains superlinear or sublinear terms. For example, the well-known Duffing system is a kind of such systems, it can be presented the sum as a linear part and a superlinear part [27], and Wang et al. [23] investigated the nonlinear control of aircraft engines by using the assumption (2.2) about the nonlinear term.

We need the following definition and lemma to prove the main results.

Definition 1 (*See [28]*)**.** For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denote the number of switchings of σ over the interval (T_1, T_2). If, for $\eta_a > 0$ and an integer $N_0 \ge 0$,

$$N_\sigma(T_1, T_2) \le N_0 + \frac{T_2 - T_1}{\eta_a},$$

then η_a is called the average dwell time and N_0 is the chattering bound.

Lemma 1. *Assume that k and p are two constants satisfying $k \ge 0$, $p > 0$, and $p \ne 1$, $u, a, b, c, d \in C(R_+, R_+)$. If*

$$u(t) \le k + \int_{t_0}^{t}\Big(a(s)u(s) + b(s)u(s-\tau(s)) + c(s)u^p(s) + d(s)u^p(s-\tau(s))\Big)ds$$

with the initial condition $u(t) = \phi(t),\ t \in [t_0 - \tau_2, t_0]$, $\phi \in C([t_0 - \tau_2, t_0], R_+)$, and $\tau(t)$ satisfies $t - \tau(t) \le t_0,\ t_0 \le t \le t^*$ and $t - \tau(t) > t_0,\ t > t^*$, where

$$t^* = \sup\{t : t - \tau(t) \le t_0\}, \tag{2.3}$$