Nonlinear Analysis: Hybrid Systems 24 (2017) 69–82

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Adaptive-observer-based output-constrained tracking of a class of arbitrarily switched uncertain non-affine nonlinear systems

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ARTICLE INFO

Article history:
Received 19 May 2016
Accepted 14 November 2016

Keywords:
Adaptive observer
Switched non-affine systems
Output constraints
Function approximation technique
Unknown control directions

ABSTRACT

This paper addresses an adaptive output-feedback tracking problem of arbitrarily switched pure-feedback nonlinear systems with time-varying output constraints and unknown control directions. In this work, the tracking problem of switched non-affine nonlinear systems with output constraints is transformed into the stabilization problem of switched unconstrained affine systems. The main contribution of this paper is to present a universal formula for constructing an adaptive state-observer-based tracking controller with only two adaptive parameters by using the common Lyapunov function method. These adaptive parameters in the proposed control scheme are derived using the function approximation technique and a priori knowledge of the signs of control gain functions is not required. The theoretical analysis is presented for the Lyapunov stability and the constraint satisfaction of the resulting closed-loop system in the presence of arbitrary switchings.

1. Introduction

In recent years, the output-constrained control problem has attracted considerable attention from the control community of nonlinear systems. In the existing results on this problem, output constraints have been considered as the predefined specifications for the transient and steady-state control performance as well as physical limitations such as saturation and physical stoppages. The barrier Lyapunov function method has been widely used to analyze the stability of the control system and the constraint satisfaction simultaneously [1]. Control design approaches using the barrier Lyapunov function have been actively presented for nonlinear systems with nonlinearities unmatched in the control input [1–5]. To deal with time-varying output constraints, recursive control designs were derived for nonlinear systems in the affine [6] and the non-affine form [7]. In addition, an output transformation method has been recently introduced for the output-constrained control design of pure-feedback nonlinear systems where time-varying constraints were regarded as the bounds of output tracking performance and the inputs of the used neural networks included all state variables [8]. Despite these efforts, the existing results [1–8] are only available for non-switched nonlinear systems with output constraints and require the measurement of full state variables.

On the other hand, intensive research activities have been reported for the control design and analysis of switched systems. These researches on switched systems are classified by multiple Lyapunov functions and common Lyapunov function methods. When the switching signal is regarded as a component of the control input, the multiple Lyapunov function method is mainly employed to design the individual controller for each switched subsystem and appropriate switching

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http://dx.doi.org/10.1016/j.nahs.2016.11.005

laws to stabilize the switched closed-loop systems [9–11]. In [12], a stabilization problem of switched linear systems with all unstable subsystems was investigated where an algorithm based on the multiple Lyapunov function method was presented to compute the stability region of admissible dwell time of the switching signal. A stabilization result of switched nonlinear systems with some unstable modes was proposed without depending on the constant ratio condition required in the dwell-time scheme [13]. In [14], a stabilization problem of switched nonlinear systems with all unstable modes was addressed where a periodical switching law was proposed to guarantee the asymptotical stability of the origin. These results require some conditions for designing switching laws and cannot guarantee the stability under arbitrary switching. On the other hand, the common Lyapunov function method has been widely utilized in the control field of switched systems with arbitrary switching signals (see [15–17] and the references therein). In particular, the backstepping technique [18] has been applied to control systems with switched nonlinearities unmatched in the control input [19–24]. To treat unknown switched nonlinearities in a lower triangular form, these research results have recently been combined with the online function approximation technique using neural networks or fuzzy systems where unknown nonlinearities were estimated by function approximators in control schemes. In [25], an adaptive neural control approach was presented for switched strict-feedback nonlinear systems. This result was extended into adaptive control designs of switched pure-feedback nonlinear systems with arbitrary switchings [26,27]. Some control approaches have been presented for switched nonlinear systems with constraints [28–31]. In [32], a robust control problem using p-times differentiable unbounded functions was investigated for uncertain switched strict-feedback nonlinear systems with tracking constraints. However, the aforementioned results [25–32] have two restrictions: (i) the results [25–32] were based on the assumption that all state variables are measurable, namely, the full state-feedback control methods were presented; and (ii) they are only available for switched nonlinear systems without constraints [25–27] or with static constraints [28–32]. To the best of our knowledge, the output-feedback control problem of arbitrarily switched non-affine nonlinear systems with time-varying output constraints is still open, and is important and challenging in both theory and real world applications. This point is a motivation of this paper.

Based on these observations, we investigate an adaptive output-feedback tracking problem of a class of uncertain switched non-affine systems with time-varying output constraints and arbitrary switchings. Switched non-affine nonlinearities unmatched in the control input and the signs of control coefficient functions are assumed to be unknown. The output transformation reported in [8] is employed to transform switched non-affine nonlinear systems with constrained outputs into switched affine nonlinear systems with unconstrained outputs. Then, the adaptive observer and controller using only an output measurement are designed via the common Lyapunov function method. The proposed common output-feedback control scheme only requires two adaptive parameters derived from the boundedness property of basis functions of radial basis function neural networks (RBFNNs). From the rigorous Lyapunov stability analysis, it is shown that the system output remains within the predefined time-varying constraints even at the moments when arbitrary switchings occur and all the signals in the closed-loop system are uniformly bounded.

Compared with the related works in the literature, the main contributions of this paper are as follows:

- (C1) The existing control result [8] using a system transformation method required full state variables as the inputs of neural networks. Thus, the *full state-feedback* tracking problem of *non-switched* nonlinear pure-feedback systems with output constraints was considered in [8]. However, this paper firstly deals with the system-transformation-based *output-feedback* tracking problem of *arbitrarily switched* nonlinear pure-feedback systems with output constraints. In particular, unlike [8], the signs of control gain functions are *unknown* in this paper; and
- (C2) Contrary to the existing full state-feedback results [25–32] for uncertain switched systems with unmatched nonlinearities, the proposed output-feedback scheme includes only two adaptive parameters and thus has a simpler structure than the control schemes presented in [25–32].
- (C3) For the stability analysis of the proposed adaptive output-feedback tracking scheme in the presence of unknown control directions, some technical lemmas are firstly developed to ensure the boundedness of Nussbaum gain functions.

2. Problem formulation

2.1. Switched non-affine nonlinear systems

Consider a class of switched non-affine nonlinear systems represented by

$$\begin{aligned} \dot{x}_i &= f_{i,\sigma(t)}(\bar{x}_i, x_{i+1}), \quad i = 1, \ldots, n-1 \\ \dot{x}_n &= f_{n,\sigma(t)}(\bar{x}_n, u_{\sigma(t)}), \\ y &= x_1, \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^\top \in \mathbb{R}^i, i = 1, \ldots, n$ are state variable vectors, y is a system output, and $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal. For any $j \in M$ and $i = 1, \ldots, n$, $u_j \in \mathbb{R}$ is a control input of the jth subsystem and $f_{i,j}(\cdot) : \mathbb{R}^{i+1} \mapsto \mathbb{R}$ are unknown continuous nonlinear functions of the jth subsystem.

Remark 1. System (1) represents state-space models of systems, i.e., some specific physical systems such as biochemical processes [18], aircraft flight control systems [33], ship maneuvering systems [34], continuous stirred tank reactor