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Robust output feedback sliding mode control for uncertain discrete time systems

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ABSTRACT

This paper proposes a robust output feedback controller for a class of uncertain discrete-time, multi-input multi-output, linear, systems. This method, which is based on the combination of discrete-time sliding mode control (*DTSMC*) and Kalman estimator, ensures the stability, robustness and an output tracking against the modeling uncertainties at large sampling periods. For this purpose, an appropriate structure is considered for sliding surface and the Lyapunov theory for the mismatched uncertain system is then used to design its parameter. This problem leads to solve a set of linear matrix inequalities. A new method is then proposed to reach the quasi-sliding mode and stay thereafter. Simulation studies show the effectiveness of the proposed method in the presence of parameter uncertainties and external disturbances at large sampling periods.

1. Introduction

In recent years, sliding mode control (*SMC*) has been extensively developed for continuous-time (*CT*) systems [1]. Its attractive features such as good tracking properties and robustness against a large class of perturbations or model uncertainties and chaotic behavior encourage the researchers to apply SMC to a variety of practical engineering systems such as robot manipulators, underwater vehicles, spacecrafts, and flexible structures [2–8].

On the other hand, due to the rapid progress of digital signal processor (*DSP*) boards and industrial computers, digital controllers have been paid more attention, recently. In this way, computers or DSP boards are used for the control of CT plants. This configuration which is named as sampled-data control systems, results in the ease of implementation, more flexibility to change, and low installation cost compared with the traditional analog control systems. However, despite such important advantages, the actual implementation of such systems, gives rise to new theoretical challenges in the SMC controller design due to coexisting of discrete-time (*DT*) and CT signals. As CT sliding mode control can maintain the states of a system on the sliding surface, discrete-time sliding mode control (*DTSMC*) try to remain the states in the vicinity of the sliding surface. This is due to the fact that control signals are generated by DTSMC at sampling instants and are held fixed over the entire sampling period. Compared with the large amount of publications on SMC for CT systems, the DTSMC is in the infancy. In order to compensate the sampling negative effects on the closed-loop performance, two basic approaches have been proposed. In the first technique which is named as digital redesign or indirect method, the DTSMC is obtained by discretization of a predesigned continuous time SMC (*CTSMC*). For this purpose, the conventional discretization methods, such as the zero-order hold (*ZOH*)-equivalent and Euler's are used; for example in [9–14]. But, in such indirect methods, the behavior of the resulting closed-loop system is not considered in the design procedure. This may lead to a periodic behavior and even unstable response in certain cases [11] and in order to preserve the closed-loop stability and tracking

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performance, a very fast sampling frequency is required [15]; a matter which cannot be guaranteed in some dynamical systems such as networked control systems [16,17]. The second approach is based on the direct design of the SMC for DT systems. In this way, the CT plant is firstly discretized using ZOH-equivalent method. The controller is then designed purely in the DT domain. In this way, some robust methods are proposed for both matched and unmatched bounded uncertainties and/or disturbances. Niu et al. [18] proposed a discrete-time sliding-mode controller for a system with matched disturbance. Pai [19] also considered the problem of robust tracking and model following for an uncertain linear system by a neural network-based DTSMC. Sun et al. [20] proposed an optimal integral sliding surface.

However, these method are limited to the systems with full-state feedback, which is not feasible in practice. For this purpose, some researchers demand the use of observers or dynamic compensators. Pai [21] considered the problem of robust tracking and model following for an uncertain linear system by an output feedback quasi-sliding mode control scheme. Zhang et al. [22] proposed an output-feedback SMC for a class of DT systems with matched disturbances. Pai [23] presented a dynamic output feedback DTSMC which is based on the radial basis function neural network. Yoshimura [24] proposed a DT adaptive sliding mode controller for uncertain systems. The above review clearly indicates that robust tracking and model following for a class of mismatched uncertain dynamical systems with partially known state information in discrete-time domain seems to be nonexistent (see Remark 2). On the other hand, from practical, as well as theoretical points of view, the existing DTSMC methods suffer from an important drawback of necessity for sampling at a high frequency. This research has been done to fill these gaps.

Motivated by the aforementioned concerns, in this paper, a *robust output feedback sliding mode control* (*ROFSMC*) strategy is proposed for robust tracking and model following of a DT linear system with mismatched parameter uncertainties which can work at low sampling frequencies. The major contributions of this work can be summarized as follows: (1) The robustness against unknown matched and mismatched parameter uncertainties and also external disturbances are guaranteed; (2) A Kalman estimator is used to estimate the current plant states. In this way, the state estimation error which is correlated with the plant dynamics, is considered as a noise with bounded norm; (3) A Lyapunov-based stability analysis is utilized to guarantee the robustness of the ROFSMC; (4) The proposed method have a good performance even with relatively low sampling frequencies; (5) The ROFSMC does not need a switching type control law. Hence, chattering phenomenon is eliminated. Simulation studies on three well-known benchmark problems demonstrate the effectiveness of the proposed method.

The remainder of this paper is organized as follows. The current section will end with introducing some notational conventions and concepts. In Section 2, problem definitions are provided. The proposed control method is described in Section 3. The effectiveness of the proposed method is demonstrated through well-known benchmark examples in Section 4.

2. Problem description

Suppose that the uncertain physical plant is *multi-input multi-output* (*MIMO*), linear system with the following state-space model,

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]u(t) + B_1 u^d(t), \tag{1a}$$

$$y(t) = Cx(t), \tag{1b}$$

where, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ and $B_1 \in \mathbb{R}^{n\times m_d}$. Here, state, input, output and bounded disturbance vectors are represented by $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^l$, $u^d(t) \in \mathbb{R}^{m_d}$, respectively. The matrices $\Delta A(t)$ and $\Delta B(t)$ denote the norm-bounded uncertainties of the system, which are assumed as follows:

$$[\Delta A(t)\ \Delta B(t)] = GF(t)[E_A\ E_B], \tag{2}$$

where G, E_A and E_B are known constant matrices of appropriate dimensions, and $F(t)$ is a time-varying matrix with Lebesgue-measurable elements satisfying $F^T(t)F(t) \le I$. It is also assumed that $u^d(t)$ is an uncorrelated white noise and CT plant (1) is controllable and observable.

Assuming a DT controller with the sampling period of T_s, ZOH-equivalent model of the plant can be illustrated as

$$x_{k+1} = (A_d + \Delta A_d)x_k + (B_d + \Delta B_d)u_k + B_{d_1}u_k^d, \tag{3a}$$

$$y_k = Cx_k. \tag{3b}$$

Here, $A_d = e^{AT_s}$, $B_d = \int_{t_k}^{t_k+T_s} e^{A(T_s-\lambda)}d\lambda B$ and $B_{d_1} = \int_{t_k}^{t_k+T_s} e^{A(T_s-\lambda)}d\lambda B_d$. In this way, it is assumed that the disturbance $u^d(t)$ does not vary too much between two consecutive sampling instances. The notations x_{k+1}, y_k, x_k, u_k and u_k^d are also used instead of $x(t_{k+1}), y(t_k), x(t_k), u(t_k)$ and $u^d(t_k)$, respectively. ΔA_d and ΔB_d are constant matrices with appropriate dimensions, representing the DT model of uncertainty.

Remark 1. In order to estimate the discrete uncertainty bound ΔA_d and ΔB_d, the following steps have been carried out [25]:

1. For sampling period T_s, discretize the unperturbed system A and B to compute A_d and B_d.
2. Select some different possible values for $F(j), j = 1, 2, \ldots, r$. According to the uncertainty bounds $\Delta A^j = GF(j)E_A$ and $\Delta B^j = GF(j)E_B$, and associated with T_s, discretize the uncertain systems $A + \Delta A^j$ and $B + \Delta B^j$, and compute A^j_{d-unc} and B^j_{d-unc}.