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Globally smooth output feedback stabilization of a class of planar switched systems with average dwell time

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ABSTRACT

Smooth output feedback stabilization for a family of planar switched nonlinear systems with average dwell time is addressed. Based on the adding a power integrator method, multiple Lyapunov functions are constructed and state stabilizing feedback laws are designed simultaneously. Multiple one-dimensional compensators for the switched nonlinear system whose subsystems may have uncontrollable/unobservable Jacobian linearization are constructed to estimate the unmeasurable states. Coupling the state feedback controllers with the nonlinear observers, globally asymptotical stabilization of the switched nonlinear system is achieved by output feedback controllers. The average dwell time method is used to choose switching signals. Simulation results are employed to verify the efficiency of the proposed method.

1. Introduction

Switched systems consist of a family of subsystems described by differential or difference equations and a switching law that orchestrates switching between these subsystems. Stability analysis and feedback stabilization of switched systems were paid much attention, especially asymptotical stability and feedback stabilization [1–9]. Stability analysis of switched systems is often implemented with common Lyapunov functions and multiple Lyapunov functions [10–15]. Common Lyapunov functions are used to discuss the stability of switched systems under arbitrary switching, but the construction of a common Lyapunov function is a hard task and there are only few results for some special systems [16,17]. Multiple Lyapunov function tools are useful in analyzing stability of slowly switched systems [1,4,18,19]. With suitable switching signals, such as dwell time and average dwell time switching signals, stability analysis can be approached by virtue of multiple Lyapunov functions.

On the other hand, asymptotical stabilization of switched nonlinear systems with special structure, especially in lower triangular form, has drawn much attention [20–26]. Under the assumption that the powers of the chained integrators are the same positive odd integer for subsystems during the whole operation time, stability analysis and state feedback stabilization are considered in [20,22–25]. In [21], globally asymptotical stability of switched nonlinear systems in p-norm where the powers of the subsystems are allowed to be different and positive even integers were discussed. For switched systems whose powers of integrators are not necessarily integers and different for different periods of the whole operation time, finite time

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state feedback stabilizers were designed in [26]. One feature of designing of the control law for the studied switched systems in the above mentioned references is all state feedback. To the best of authors' knowledge, output feedback stabilization of switched nonlinear systems is still a challenging task [27].

Global output feedback stabilization is one of the most fundamental problems in the area of nonlinear control. Usually, the separation principle does not hold for nonlinear system anymore. Hence, output feedback stabilization of nonlinear system is much more challenging in the large compared with global stabilization via state feedback. In the past few decades, this difficult problem has been paid much attentions and some interesting results were obtained. Being aware of the fact that suitable growth conditions are necessary [28], output feedback stabilization of nonlinear systems under various structural or growth conditions, such as feedback equivalence to an output feedback form [29,30] or a triangular form with certain conditions, linear or global Lipschitz-like condition on the unmeasurable states [31,32], has been focused on. The listed results above are established under the condition that the nonlinear system has a controllable and observable linearization, at least partially. For the inherent nonlinear systems whose first approximation is neither controllable nor observable, an approach called adding a power integrator has been used to study global output feedback stabilization of the system, such as smooth output feedback stabilization of planar systems [33], nonsmooth output feedback stabilization of nonlinear systems [34] and global output feedback stabilization of nonlinear systems via sampled-data control [35].

As has been stated, in the case of asymptotical stabilization of switched nonlinear systems by virtue of state feedback control law, there were some elegant works, such as the results in Refs. [20–26]. Based on the assumption that for every subsystem there is a control Lyapunov function (Assumption 1) and some observable conditions (Assumption 2), output feedback controllers and switching logic are presented in [27]. The disadvantage of the method is that it does not yield any clue on how to find the control Lyapunov function. Moreover, if the subsystem is not observable any more, the method may be fail. Due to the switchings between different subsystems whose linearization may be uncontrollable or unobservable, output feedback stabilization of switched nonlinear systems is a far more difficult issue than that via state feedback or output feedback with some observable conditions. In this note, based on the approach called adding a power integrator, globally asymptotical stabilization by output feedback control for the planar switched nonlinear system is discussed. Our contributions are as follows: (1) Multiple Lyapunov functions are constructed and state feedback stabilizing laws are presented simultaneously based on adding a power integrator method; (2) Multiple one-dimensional nonlinear observers are designed to estimate the unmeasurable states; (3) Output feedback controllers are designed to stabilize the planar switched nonlinear system, coupling the state feedback controllers with the proposed nonlinear observers; (4) Switching signals is given by virtue of average dwell time method to guarantee the stability of switched nonlinear system.

The paper is organized as follows. Preliminaries and system description are introduced in Section 2. In Section 3, a systematic design procedure is presented. Illustrative examples are given in Section 4. Finally, some conclusions are drawn in Section 5.

2. Preliminaries and problem formulation

In this paper, we consider a switched nonlinear system described by equations of the form

$$\begin{cases}\dot{x}_1 = x_2^{p_i} + \phi_{\sigma(t),1}(x_1)\\ \dot{x}_2 = u_i + \phi_{\sigma(t),2}(x_1, x_2)\end{cases} \tag{1}$$

where $x = (x_1, x_2)^T$ is the system state, $u \in \mathbb{R}$ is the control input, and $p_i \geq 1$ are odd integers, $\sigma(t) = i \in S_N = \{1, 2, \ldots, M\}$, M is the number of the subsystems of switched system (1).

Two assumptions are given as follows.

Assumption 1. For $\sigma(t) = i \in S_N$, there are nonnegative constants $c_{i,1}, c_{i,2}$ such that

$$\begin{aligned}&|\phi_{i,1}(x_1)| \leq c_{i,1}|x_1|^{p_i}\\ &|\phi_{i,2}(x_1, x_2)| \leq c_{i,2}(|x_1|^{p_i} + |x_2|^{p_i}).\end{aligned} \tag{2}$$

Assumption 2. Switched nonlinear system (1) has finite number of switchings on any finite interval of time.

Assumptions 1 and 2 are standard assumptions. In the literature, Assumption 1 represents an important class of nonlinear systems which is commonly used in [33,34] and Assumption 2 is aimed to rule out Zeno behavior for all types of switching [4].

The relevant stability concepts of switched systems, which are the appropriately modified versions, are listed as follows.

Definition 1 ([4]). Consider the family of switched nonlinear systems $\dot{x} = f_i(x)$, where $x \in R^n$, $f_i(x)$ is deferential continuous, $i \in S_N$. It is said to be asymptotically stable if there exist a positive constant δ and a class KL function $\beta(\cdot, \cdot)$ such that for switching signals $\sigma(t)$ the solutions of (1) with $|x(0)| \leq \delta$ satisfy the inequality

$$|x(t)| \leq \beta(|x(t)|, t), \quad \forall t \geq 0. \tag{3}$$