Nonlinear Analysis: Hybrid Systems 24 (2017) 198–226

Contents lists available at ScienceDirect

Nonlinear Analysis: Hybrid Systems

journal homepage: www.elsevier.com/locate/nahs

Dwell-time stability and stabilization conditions for linear positive impulsive and switched systems

Corentin Briat

Department of Biosystems Science and Engineering, ETH Zürich, Switzerland

ARTICLE INFO

Article history:
Received 16 March 2016
Accepted 17 January 2017

Keywords:
Positive systems
Impulsive systems
Switched systems
Clock-dependent conditions

ABSTRACT

Several results regarding the stability and the stabilization of linear impulsive positive systems under arbitrary, constant, minimum, maximum and range dwell-time are obtained. The proposed stability conditions characterize the pointwise decrease of a linear copositive Lyapunov function and are formulated in terms of finite-dimensional or semi-infinite linear programs. To be applicable to uncertain systems and to control design, a lifting approach introducing a clock-variable is then considered in order to make the conditions affine in the matrices of the system. The resulting stability and stabilization conditions are stated as infinite-dimensional linear programs for which three asymptotically exact computational methods are proposed and compared with each other on numerical examples. Similar results are then obtained for linear positive switched systems by exploiting the possibility of reformulating a switched system as an impulsive system. Some existing stability conditions are retrieved and extended to stabilization using the proposed lifting approach. Several examples are finally given for illustration.

1. Introduction

Linear positive systems [1] have been recently the subject of an increasing attention because of their natural ability to represent many real-world processes such as, among others, communication networks [2,3], biological networks [4–7], epidemiological networks [4,8], and disease dynamics [9]. Besides their applicative potential, linear positive systems have been shown to exhibit a number of interesting theoretical properties of independent interest. For instance, it is now well-known that linear copositive Lyapunov functions can be used in order to formulate exact stability conditions taking the form of linear programs [10]. The design of structured and bounded state-feedback controllers [11,12] and certain classes of static output feedback controllers [13] are known to be convex and hence easily tractable. The L_p-gains for $p = 1, 2, \infty$ can be exactly computed using convex programming and these gains are identical to the p-norm of the static matrix-gain of the system [14,15]. The famous Kalman–Yakubovich–Popov Lemma has been shown to admit a linear formulation in this setting [16]. Robust analysis results also nicely extend and simplify in this context, and often lead to necessary and sufficient criteria for stability [12,15,17–19]. Their generalization to delay-systems with discrete-delays also led to the surprise that the system is stable if and only if the system with zero delay is stable [12,20–22]. Extensions to deterministically [23–26] or stochastically [27,28] switched systems have also been considered. Positive systems have also been recently used as (conservative) comparison systems for establishing the stability of various classes of systems such as systems with delays [29–32]. Finally, the design of interval observers heavily relies on the use of positive systems theory [33–35]. It was notably shown in [35] that the observer-gain that minimizes the L_∞-gain of map between the disturbance input and the

E-mail addresses: corentin.briat@bsse.ethz.ch, corentin@briat.info.
URL: http://www.briat.info.

http://dx.doi.org/10.1016/j.nahs.2017.01.004

observation error is independent of the output matrices of the error system, a result that mirrors that of [36] pertaining on the state-feedback control of linear positive systems achieving minimum L_1-gain.

We consider here the case of linear positive impulsive systems, a class of systems that seem to have been quite overlooked until now as only very few results can be found; see e.g. [37–39]. Such systems can be used to represent certain classes of biochemical, population or epidemiological models having deterministic jumps in their dynamics. They can also be used to represent processes that can be represented as linear positive switched systems; see e.g. [26] for some examples including epidemiology [40–42], traffic congestion models [43], etc. Impulsive systems are also known to be able to exactly represent sampled-data systems as emphasized in [44–47]. Such systems are also interesting from a theoretical standpoint as they can be useful for the analysis and design of interval observers for linear impulsive systems (and hence sampled-data and switched systems) or for analyzing the stability of nonlinear impulsive, switched and sampled-data systems; see e.g. [37].

The goal of this paper is hence the derivation of novel stability and stabilization conditions for linear positive impulsive systems using the concepts of arbitrary, constant, minimum, maximum and range dwell-times. The concept of *minimum dwell-time* has been introduced by Morse in [48] in order to formulate stability conditions for general (i.e. not necessarily positive) switched systems. The concept of *average dwell-time* has been proposed in [49] in order to obtain less conservative conditions than by using minimum dwell-time conditions. Since then, a large body of the literature has been focusing on these concepts as a way to efficiently characterize the stability of switched systems or, more generally, the stability of hybrid systems; see e.g. [50]. The notion of minimum dwell-time has been revisited in [51] where novel sufficient LMI conditions derived from mode-dependent quadratic Lyapunov functions were proposed. Based on a theoretical result proved in [52], these conditions were later extended and made necessary and sufficient in [53] through the consideration of mode-dependent homogeneous Lyapunov functions. Analogous results using polyhedral Lyapunov functions have been also obtained in [54]. Unfortunately, these conditions were inapplicable to uncertain systems and to control design because of their complex nonlinear dependency in the matrices of the system. This problem motivated the introduction of the so-called *looped-functionals*, a particular class of indefinite (i.e. not necessarily positive definite) functionals having the advantage of reformulating the complex conditions of [51] into conditions being affine/convex in the matrices of the system; see e.g. [55–57], thereby extending the scope of the conditions to uncertain and nonlinear systems. Yet, these conditions were difficult to apply in the context of control design because of the presence of multiple products between decision matrices and the matrices of the system; see e.g. [56–59]. *Clock-dependent conditions* have been shown to provide an essential framework for solving this latter problem as they produce stability conditions that are affine/convex in the matrices of the system which can be used for design purposes. Their computational complexity has also been shown to be much lower than that of looped-functionals [59]. Since then, clock-dependent conditions have been used for the analysis and control of switched, impulsive, sampled-data and LPV systems; see e.g. [47,60–68]. Such results have also been applied to more practical problems such as fault tolerant control [69,70] or estimation [62,71].

The first part of the paper is similar to the ones in [47,72] where stability conditions are formulated in terms of the decrease of a Lyapunov function of a given type. Unlike in the previous references where quadratic Lyapunov functions are involved, we exploit here the positivity of the system and consider linear copositive Lyapunov functions [10]. The resulting conditions are stated in terms of finite-dimensional or semi-infinite dimensional linear programs, which are then relaxed into clock-dependent conditions using a lifting approach similar to that of [47,59,65,66]. Since linear copositive Lyapunov functions are used here, the clock-dependent conditions consist of infinite-dimensional linear programs. This has to be contrasted with the fact that, when quadratic Lyapunov functions are used, clock-dependent conditions take the form of infinite-dimensional semidefinite programs, which may be harder to solve that their linear counterpart. Three possible ways for efficiently checking these conditions are then proposed. The first one relies on a discretization approach which is largely inspired from [60] and where the infinite-dimensional decision variable is assumed to be continuous and piecewise linear. This method has also been considered, in turn, in [59,61,62,64,67,73]. By doing so, the infinite-dimensional program becomes finite-dimensional and can be solved using conventional algorithms such as interior point methods; see e.g. [74]. The second method is based on Handelman's theorem [75] which characterizes the positivity of a given polynomial on a compact polytope by formulating it as a nonnegative linear combination of products of the (affine) basis functions that describe the polytope. This result has been applied in various contexts [12,15,76,77] and, notably, for characterizing the robust stability of uncertain linear positive systems in [12,15]. An important property of this approach is that the obtained characterization for the positivity of the polynomial can be exactly formulated as a finite-dimensional linear program, which can again be solved using well-known approaches. Finally, the last one is based on Putinar's Positivstellensatz [78] which characterizes the positivity of a given polynomial on a compact semialgebraic set by formulating it as a weighted linear combination of the basis functions that describe the set and where the weights are sum of squares polynomials. The resulting problem takes, in this case, the form of a finite-dimensional semidefinite program [79] that can be solved using standard semidefinite programming solvers such as SeDuMi [80] or SDPT3 [81] used in conjunction with the package SOSTOOLS [82]. It is notably emphasized that these relaxations are asymptotically exact meaning that when the discretization order, the number of products of basis functions or the degree of the sum of squares weights are sufficiently large, then the relaxed problem is feasible if the original one is. Several examples are considered in order to demonstrate the practicality of the relaxed conditions and to compare them in terms of number of variables and solving time. The results are then extended to control design by considering the clock-dependent conditions and the dual impulsive system [26,83]. By finally exploiting the possibility of formulating a switched system as an impulsive system, we derive a number of stability conditions for linear positive switched systems. Notably, we recover stability conditions similar to those in [25,26] which are the positive