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A Cartesian relative motion approach to optimal formation flight using Lorentz forces and impulsive thrusting

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ABSTRACT

Hybrid combination of Lorentz forces and impulsive thrusts, provided by modulating spacecraft's electrostatic charge and propellant usage, respectively, is proposed for formation flight applications. A hybrid linear quadratic regulator, previously proposed in another work using a differential orbital elements-based model, is reconsidered for a Cartesian coordinates-based description of the spacecraft's relative states. In addition, the effects of adopting circular versus elliptic reference solutions on the performance of the controller are studied. Numerical simulation results are provided to demonstrate the functionality of the proposed controller in the presence of J_2 perturbations, and to illustrate the improvements gained by assuming an elliptic reference and incorporating auxiliary impulsive thrusts.

1. Introduction

Formation flight of spacecraft, involving groups of multiple satellites that orbit in proximity of each other, has seen a lot of renewed interest in recent years. This is particularly because of the improvements they offer over single-spacecraft missions in terms of affordability and robustness, and is facilitated by recent technological and scientific developments that enable reliable formation missions. One potential approach for spacecraft to achieve and maintain formation is via Lorentz-augmented control. The idea of using Lorentz forces generated by the interaction of actively-modulated charges on a spacecraft with the geomagnetic field in order to produce useful thrust was first proposed in Ref. [1].

In Ref. [2], analytical solutions of the equations of motion for Lorentz-augmented spacecraft in various situations were provided; however, only constant specific charges and circular reference frames were considered. The equations of motion linearized relative to a circular reference orbit were presented in Refs. [3,4], and are known as the Hill-Clohessy-Wiltshire (HCW) equations. Also using a spherical co-ordinates description similarly to [2], a three-spacecraft formation reconfiguration problem was considered in Ref. [5], but assuming proportional derivative-type feedback control provided by modulating the specific charge. Abandoning the circularity assumption on the chief spacecraft's orbit, Ref. [6] considered both circular and elliptic references using Cartesian coordinates for relative motion. In that work, step-wise charge control based on the linearized model, as well sequential quadratic programming using the nonlinear model were proposed. The relative motion equations that allow for elliptic reference orbits are known as Tschauner-Hempel (TH) equations, and were provided in Refs. [7,8], among others.

In contrast to the use of spherical or Cartesian coordinates to describe the relative motion of spacecraft in formation, an alternative is to focus on the changes in the mean orbital elements, hence ignoring the short-term oscillations. Examples of past literature that make use of (mean) orbital elements (or their differences) for formation control are [9,10], and those of works that involve Lorentz-augmentation in particular are [11,12]. This approach is primarily motivated by the fact that, in many formation flight missions, only secular changes are of importance when determining tracking errors. While recognizing the value of this approach (especially in the presence of J_2 perturbations), the present authors have chosen to work directly with the Cartesian description of the spacecraft's absolute and relative positions. It is expected that such an approach will be better suited to applications for which short-term errors do matter. In order to demonstrate the effectiveness of the proposed controller and its comparability with the mean orbital elements-based techniques, J_2 influences are modelled in all simulation results to be presented. It is shown that the required specific charge and thrust magnitudes are still reasonable.

Hybrid formation control of spacecraft using continuous and impulsive forces in tandem is considered in this paper, based on a Cartesian coordinates-based model, and the methodology is applicable to both

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circular and elliptic reference orbits for the chief spacecraft. The work presented in Ref. [6] considered the in-plane and out-of-plane motions separately when using the linearized model, and resorted to numerical approaches based on nonlinear trajectory design in order to treat more general cases with smaller errors. On the other hand, explicit expressions for the continuous and impulsive forces are presented in this document (although the associated Riccati equation still needs to be integrated numerically), and the complete three-dimensional motion is treated in a unified manner. The reference solution presented in Ref. [13] for elliptic orbits, in turn built upon the TH equations as presented in Ref. [8], are adopted in this work.

As demonstrated in Refs. [14,15] by studying the system's Gramian matrix, there is always one direction along which Lorentz-augmented formation is not controllable. This is because the Lorentz force is always perpendicular to the plane defined by the geomagnetic field vector and the spacecraft's velocity relative to the field. Similarly, controllability in the in-plane motion of equatorial reference orbits was demonstrated in Ref. [6], but out-of-plane motion was initially ignored in that analysis (and later on treated as uncontrolled drift). Motivated by a desire to overcome this controllability issue, the present work features a hybrid formulation that combines continuous-time Lorentz forces with impulsive thrusts, a problem that was treated in Refs. [12,14] based on a mean orbital elements-based model (as opposed to the current Cartesian formulation), as well as in Ref. [16]. As opposed to [16] that used trajectory optimization techniques and the pseudo-spectral method (followed by *a posteriori* distribution of the required control accelerations into continuous and impulsive contributions), the work described here uses a hybrid linear quadratic regulator (LQR) scheme based on that used in Refs. [12,14].

The primary rationale behind using LQR in the present work is that it is a well-established optimal control method that allows the user to trade off control effort against state errors. It also lends itself well to a hybrid formulation and the associated optimization, as delineated in the aforementioned references. Use of LQR in the context of formation flight was also seen in Ref. [17], assuming a discrete-time system and limiting the study to in-plane motion; in Ref. [18], using low-thrust continuous forces (not Lorentz forces) with circular reference orbits and accommodating gravitational disturbances; and in Ref. [19], also using continuous forces but allowing for the reference orbit's ellipticity.

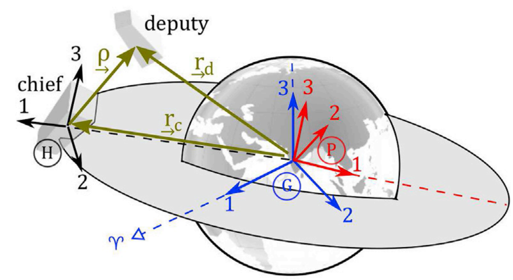
The organization of this paper is as follows. A mathematical model of formation flight subject to Lorentz and thruster forces is constructed in Section 2 using relative motion equations in Cartesian coordinates. The hybrid LQR scheme to be used for control purposes is described in Section 3, along with circular and elliptic reference orbit solutions to be adopted. The functionality and performance of the proposed hybrid controller are demonstrated in Section 4 via numerical simulations, and the effects of reference orbit selection and incorporating auxiliary impulsive thrust on the performance are studied. Lastly, concluding remarks are made in Section 5.

2. Mathematical modelling of formation flight

This section establishes the mathematical model of the motion, to be used for control purposes in Section 3. The relationships that describe the geometry of motion are presented in Subsection 2.1, and the motion equations in the presence of applied forces are provided in Subsection 2.2.[1]

2.1. Kinematics

The following reference frames, illustrated in Fig. 1, are defined and used throughout the document:



(Earth Vector Graphic by WebDesignHot)

Fig. 1. Reference frames: (G) ECI, (P) perifocal, and (H) Hill.

- $\mathscr{F}_G$ – Earth-Centred Inertial (ECI) frame: origin at Earth's centre, 1-axis towards the vernal equinox, 3-axis towards Earth's North pole
- $\mathscr{F}_P$ – Perifocal frame: origin at Earth's centre, 1-axis towards the perigee of the chief's orbit, 3-axis normal to the chief's orbital plane
- $\mathscr{F}_H$ – Hill frame: origin at the chief's centre of mass, 1-axis pointing away from Earth's centre, 3-axis normal to the chief's orbital plane

The position vectors, measured from Earth's centre, of the chief and the deputy are given by $\underset{\rightarrow}{\mathbf{r}}_c$ and $\underset{\rightarrow}{\mathbf{r}}_d$, respectively, the components of which in the ECI frame are given by column matrices, $\mathbf{r}_{c,G}$ and $\mathbf{r}_{d,G}$. Rotation matrices can then be used to obtain their corresponding Hill frame representations:

$$\mathbf{r}_{c,H} = \mathbf{C}_{HG}\mathbf{r}_{c,G} \quad , \quad \mathbf{r}_{d,H} = \mathbf{C}_{HG}\mathbf{r}_{d,G} \tag{1}$$

where $\mathbf{C}_{HG}$ is the rotation matrix from $\mathscr{F}_G$ to $\mathscr{F}_H$ which can be evaluated, along with its rate of change, as follows [19]:

$$\mathbf{C}_{HG} = \begin{bmatrix} \dfrac{\mathbf{r}_{c,G}}{|\mathbf{r}_{c,G}|} & \dfrac{\mathbf{h}^{\times}_{c,G}\mathbf{r}_{c,G}}{|\mathbf{h}^{\times}_{c,G}\mathbf{r}_{c,G}|} & \dfrac{\mathbf{h}_{c,G}}{|\mathbf{h}_{c,G}|} \end{bmatrix}^{\mathsf{T}} \tag{2a}$$

$$\dot{\mathbf{C}}_{HG} = \begin{bmatrix} \dfrac{d}{dt}\left(\dfrac{\mathbf{r}_{c,G}}{|\mathbf{r}_{c,G}|}\right) & \dfrac{d}{dt}\left(\dfrac{\mathbf{h}^{\times}_{c,G}\mathbf{r}_{c,G}}{|\mathbf{h}^{\times}_{c,G}\mathbf{r}_{c,G}|}\right) & \dfrac{d}{dt}\left(\dfrac{\mathbf{h}_{c,G}}{|\mathbf{h}_{c,G}|}\right) \end{bmatrix}^{\mathsf{T}} \tag{2b}$$

where $\mathbf{h}_{c,G} = \mathbf{r}^{\times}_{c,G}\mathbf{v}_{c,G}$ denotes the ECI representation of the chief's orbital angular momentum vector. The derivatives in Eq. (2b) can be evaluated using the following identity for a generic $\mathbf{x}$:

$$\frac{d}{dt}\left(\frac{\mathbf{x}}{|\mathbf{x}|}\right) = \frac{\dot{\mathbf{x}}}{|\mathbf{x}|} - \frac{|\dot{\mathbf{x}}|}{|\mathbf{x}|^2}\mathbf{x} = \frac{\dot{\mathbf{x}}}{|\mathbf{x}|} - \frac{\mathbf{x}^{\mathsf{T}}\dot{\mathbf{x}}}{|\mathbf{x}|^3}\mathbf{x} \tag{3}$$

in conjunction with the following known relationships:

$$\frac{d}{dt}(\mathbf{r}_{c,G}) = \dot{\mathbf{r}}_{c,G} = \mathbf{v}_{c,G} \tag{4a}$$

$$\frac{d}{dt}(\mathbf{h}_{c,G}) = \dot{\mathbf{h}}_{c,G} = \mathbf{r}^{\times}_{c,G}\ddot{\mathbf{r}}_{c,G} = \mathbf{r}^{\times}_{c,G}\mathbf{f}_{p,G}(\mathbf{r}_{c,G}) \tag{4b}$$

$$\frac{d}{dt}\left(\mathbf{h}^{\times}_{c,G}\mathbf{r}_{c,G}\right) = \dot{\mathbf{h}}^{\times}_{c,G}\mathbf{r}_{c,G} + \mathbf{h}^{\times}_{c,G}\dot{\mathbf{r}}_{c,G} \tag{4c}$$

where $\mathbf{v}_{c,G}$ and $\mathbf{f}_{p,G}$ consist of the ECI components of the chief's absolute velocity vector and the perturbation forces (per unit mass) experienced by the chief. Lastly, noting that the spacecraft's relative position is given by $\boldsymbol{\rho} \triangleq \mathbf{r}_d - \mathbf{r}_c$, taking the difference of the position vectors in Eq. (1) and differentiating with respect to time yields:

$$\boldsymbol{\rho}_H = \mathbf{C}_{HG}\boldsymbol{\rho}_G \tag{5a}$$

$$\dot{\boldsymbol{\rho}}_H = \dot{\mathbf{C}}_{HG}\boldsymbol{\rho}_G + \mathbf{C}_{HG}\dot{\boldsymbol{\rho}}_G \tag{5b}$$

[1] See Nomenclature at the end of the paper.