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Gravitational force and torque on a solar power satellite considering the structural flexibility

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ABSTRACT

The solar power satellites (SPS) are designed to collect the constant solar energy and beam it to Earth. They are traditionally large in scale and flexible in structure. In order to obtain an accurate model of such system, the analytical expressions of the gravitational force, gravity gradient torque and modal force are investigated. They are expanded to the fourth order in a Taylor series with the elastic displacements considered. It is assumed that the deformation of the structure is relatively small compared with its characteristic length, so that the assumed mode method is applicable. The high-order moments of inertia and flexibility coefficients are presented. The comprehensive dynamics of a large flexible SPS and its orbital, attitude and vibration evolutions with different order gravitational forces, gravity gradient torques and modal forces in geosynchronous Earth orbit are performed. Numerical simulations show that an accurate representation of the SPS′ dynamic characteristics requires the retention of the higher moments of inertia and flexibility. Perturbations of orbit, attitude and vibration can be retained to the 1-2nd order gravitational forces, the 1-2nd order gravity gradient torques and the 1-2nd order modal forces for a large flexible SPS in geosynchronous Earth orbit.

1. Introduction

The solar power satellites (SPS) are designed to collect the solar energy and electromagnetically beam it to Earth. The SPS mainly consists of a solar energy collector and a large antenna which beams the microwave power to the ground. Glaser first presented the Solar Power Satellite concept in 1968 [1,2]. The main benefits of collecting sunlight in space are that solar energy is not attenuated by Earth's atmosphere or climate, collection is not influence by the day-night cycle, and power may be directly transmitted to the ground. Previous studies suggest the most effective orbital location for the operation of SPS structures is in equatorial geostationary Earth orbit [3,4]. This ensures the whole day power supply with only small outages around the equinoxes. What's more, a single kilometer-wide band of geosynchronous Earth orbit experiences enough solar flux in one year (approximately 212 TW-years) to nearly equal the amount of energy contained within all known recoverable conventional oil reserves on Earth today (approximately 250 TW-yrs) [5].

In the past decades, the research focuses of the SPS are energy transmission by microwave and the configuration design. One of the most challenging technologies for the SPS is microwave power transmission from the geostationary orbit to the ground, and has been studied for more than 40 years [6]. For the configurations and research projects of the SPS, different SPS concepts have been proposed like the novel SPS concept with spherical condenser based on ε-near-zero metamaterial [7]. Many institutes led by the United States, Europe and Japan, have also carried out some conceptual researches on the SPS. A series of research projects has been proposed. Such as the 1979 SPS Reference Concept defined by the US Department of Energy and NASA [8], JAXA's SPS 2000 [9], NASA's Integrated Symmetrical Concentrator [10], European sail tower SPS concept [11], and the SPS-ALPHA [12].

Since the size of the SPS is usually in the order of kilometers, it leads to the huge mass and inertia, and strong flexibility. The system dynamic characters are different from normal satellites. Therefore, more attentions should be paid on the dynamics and control the attitude, orbital, and structural motion of this large structure. The first challenge posed by large space structure control is the design of control systems with natural frequencies above several major structural frequencies [13]. McNally found that when both attitude and orbit control are considered, attitude dynamics of SPS in the Geosynchronous Laplace Plane Orbit can save fuel compared to in GEO [14]. He also performed the details of orbital dynamics of SPS, determined the long-term orbital behavior, and found an

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Nomenclature

$\boldsymbol{A}_{be}$	rotation matrix taking the inertial frame F_e to F_b
$\boldsymbol{D}$	modal damping matrix
$\vec{F}_G$, $\vec{F}_{Gi}$	total gravitational force and its components of order i
$\underset{\rightarrow}{F_m}$, $\underset{\rightarrow}{F_{mi}}$	total modal force and its components of order i
$\boldsymbol{I}^1$, $\boldsymbol{I}^{2A}$, $\boldsymbol{I}^{2B}$, $\boldsymbol{I}^{3A}$, $\boldsymbol{I}^{3B}$, $\boldsymbol{I}^{4AA}$, $\boldsymbol{I}^{4AB}$, $\boldsymbol{I}^{4BB}$	A and B (and related components) of the first, second, third and fourth moments of inertia about $\vec{\rho}'$
$\boldsymbol{I}_m^{1a}$, $\boldsymbol{I}_m^{1b}$, $\boldsymbol{I}_m^{2A}$, $\boldsymbol{I}_m^{2Ba}$, $\boldsymbol{I}_m^{2Bb}$, $\boldsymbol{I}_m^{3Aa}$, $\boldsymbol{I}_m^{3Ab}$, $\boldsymbol{I}_m^{3Ba}$, $\boldsymbol{I}_m^{3Bb}$, $\boldsymbol{I}_m^{4AA}$, $\boldsymbol{I}_m^{4ABa}$, $\boldsymbol{I}_m^{4ABb}$, $\boldsymbol{I}_m^{4BBa}$, $\boldsymbol{I}_m^{4BBb}$	A and B (and related components) of the first, second, third and fourth modal force coefficients about $\vec{\rho}'$
$\boldsymbol{J}^1$, $\boldsymbol{J}^{2A}$, $\boldsymbol{J}^{2B}$, $\boldsymbol{J}^{3A}$, $\boldsymbol{J}^{3B}$, $\boldsymbol{J}^{4AA}$, $\boldsymbol{J}^{4AB}$, $\boldsymbol{J}^{4BB}$	A and B (and related components) of the first, second, third and fourth moments of inertia about $\vec{\rho}$
$\boldsymbol{J}_m^{1a}$, $\boldsymbol{J}_m^{1b}$, $\boldsymbol{J}_m^{2A}$, $\boldsymbol{J}_m^{2Ba}$, $\boldsymbol{J}_m^{2Bb}$, $\boldsymbol{J}_m^{3Aa}$, $\boldsymbol{J}_m^{3Ab}$, $\boldsymbol{J}_m^{3Ba}$, $\boldsymbol{J}_m^{3Bb}$, $\boldsymbol{J}_m^{4AA}$, $\boldsymbol{J}_m^{4ABa}$, $\boldsymbol{J}_m^{4ABb}$, $\boldsymbol{J}_m^{4BBa}$, $\boldsymbol{J}_m^{4BBb}$	A and B (and related components) of the first, second, third and fourth modal force coefficients about $\vec{\rho}$
$\boldsymbol{P}^1$, $\boldsymbol{P}^{2A}$, $\boldsymbol{P}^{2B}$, $\boldsymbol{P}^{3A}$, $\boldsymbol{P}^{3B}$, $\boldsymbol{P}^{4AA}$, $\boldsymbol{P}^{4AB}$, $\boldsymbol{P}^{4BB}$	A and B (and related components) of the first, second, third and fourth moments of inertia about $\vec{u}$
$\boldsymbol{P}_m^{1a}$, $\boldsymbol{P}_m^{1b}$, $\boldsymbol{P}_m^{2A}$, $\boldsymbol{P}_m^{2Ba}$, $\boldsymbol{P}_m^{2Bb}$, $\boldsymbol{P}_m^{3Aa}$, $\boldsymbol{P}_m^{3Ab}$, $\boldsymbol{P}_m^{3Ba}$, $\boldsymbol{P}_m^{3Bb}$, $\boldsymbol{P}_m^{4AA}$, $\boldsymbol{P}_m^{4ABa}$, $\boldsymbol{P}_m^{4ABb}$, $\boldsymbol{P}_m^{4BBa}$, $\boldsymbol{P}_m^{4BBb}$	A and B (and related components) of the first, second, third and fourth modal force coefficients about $\vec{u}$
$\vec{r}_0$, $\vec{r}$	position vector from the Geocentric to the origin of F_b, to an elementary mass
$\boldsymbol{T}$, $\boldsymbol{T}_i$	modal matrix and its components of order i
$\vec{T}_G$, $\vec{T}_{Gi}$	total gravity gradient torque and its components of order i
$\boldsymbol{U}^{2A}$, $\boldsymbol{U}^{2Bi}$, $\boldsymbol{U}^{3Ai}$, $\boldsymbol{U}^{3Bi}$, $\boldsymbol{U}^{4AAi}$, $\boldsymbol{U}^{4ABi}$, $\boldsymbol{U}^{4BBi}$	A and B (and related components) of the second, third and fourth moments of inertia about $\vec{\rho}$ and $\vec{u}$ coupled
$\boldsymbol{U}_m^{2A}$, $\boldsymbol{U}_m^{2Bai}$, $\boldsymbol{U}_m^{2Bbi}$, $\boldsymbol{U}_m^{3Aai}$, $\boldsymbol{U}_m^{3Abi}$, $\boldsymbol{U}_m^{3Bai}$, $\boldsymbol{U}_m^{3Bbi}$, $\boldsymbol{U}_m^{4AAi}$, $\boldsymbol{U}_m^{4ABai}$, $\boldsymbol{U}_m^{4ABbi}$, $\boldsymbol{U}_m^{4BBai}$, $\boldsymbol{U}_m^{4BBbi}$	A and B (and related components) of the second, third and fourth modal force coefficients about $\vec{\rho}$ and $\vec{u}$ coupled
$\vec{u}$	deformation of an elementary mass in F_b
$\boldsymbol{v}_0$	velocity of the origin of F_b with respect to F_e
$\overleftrightarrow{\delta}$	unit dyadic
ε	ratio of characteristic spacecraft dimension to orbital radius
$\boldsymbol{\Lambda}$	modal stiffness matrix
μ	Earth's gravitational parameter
$\vec{\rho}$	position vector of an elementary mass from the origin of F_b
$\vec{\rho}'$	position vector of an elementary mass after deformation from the origin of F_b
$\boldsymbol{\tau}$, τ_i	modal coordinate matrix and its components of order i
$\boldsymbol{\omega}$	angular velocity of F_b with respect to F_e

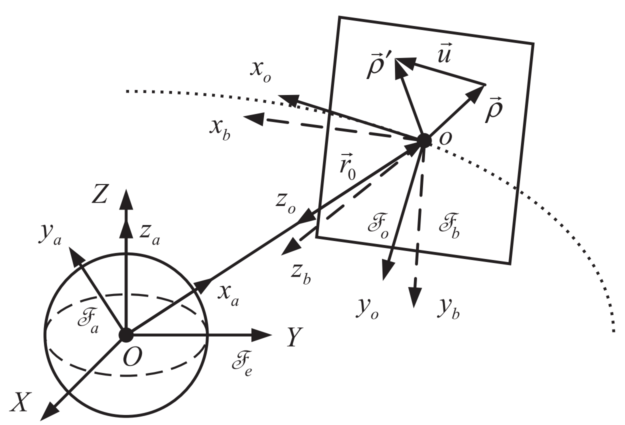

Fig. 1. Geometric position and elastic deformation of the SPS.

alternative orbital location of SPS, known as the geosynchronous Laplace plane, which is superior to geostationary orbit in many aspects [15]. Wie developed the concepts for controlling orbit, attitude, and structural motions of the very large SPS. He focused on a 1.2-GW "Abacus" SPS configuration, derived a complete set of mathematical models including solar radiation pressure, microwave radiation, gravity gradient torque, and other orbit perturbation effects, and then utilized the cyclic-disturbance accommodating control to provide ±5 arcmin pointing of the "Abacus" platform with distributed electric propulsion thrusters [16,17]. Wu first proposed the sun-pointing dynamics for a huge rigid SPS and designed a LQR controller to assess the pointing accuracy [18]. For the actuator saturation issue of engineering structure, Peng proposed a novel fast model predictive control with actuator saturation for large-scale structures [19]. He also developed two practical model predictive control implementation algorithms with multi-input delay in discrete-time formulation for vibration control of large-scale structures, which improved the computation efficiency and the memory requirement [20]. However, the system is assumed to be rigid in these preliminary studies.

Table 1
The first order coefficients.

High-order constants	Moments of inertia	Modal force coefficients
–	$\boldsymbol{J}^1 = \int \vec{\rho}\,\mathrm{d}m$	$\boldsymbol{J}_m^{1a} = \int \underset{\rightarrow}{\vec{T}} \cdot \vec{\rho}\,\overleftrightarrow{\delta}\,\mathrm{d}m = \boldsymbol{\Phi}^{23}$
–	–	$\boldsymbol{J}_m^{1b} = \int \underset{\rightarrow}{\vec{T}}\,\vec{\rho}\,\mathrm{d}m = \boldsymbol{\Phi}^{24}$
$\boldsymbol{\Phi}^1 = \int \vec{T}_{\rightarrow}\mathrm{d}m$	$\boldsymbol{P}^1 = \int \vec{u}\,\mathrm{d}m = \boldsymbol{\Phi}^{1} * \underset{\rightarrow}{\tau}$	$\boldsymbol{P}_m^{1a} = \int \underset{\rightarrow}{\vec{T}} \cdot \vec{u}\,\overleftrightarrow{\delta}\,\mathrm{d}m = \boldsymbol{\Phi}^{21} * \tau_{\rightarrow}$
–	–	$\boldsymbol{P}_m^{1b} = \int \underset{\rightarrow}{\vec{T}}\,\vec{u}\,\mathrm{d}m = \boldsymbol{\Phi}^{22} * \underset{\rightarrow}{\tau}$

On the other hand, due to the large size of the SPS, it is necessary to study the influence of high-order terms of gravitational force and gravity gradient torque. Hughes used the Taylor series to expand the gravitational force and torque to the fourth order, and introduced higher moments of inertia [21]. Ashenberg considered a class of problems in which it is impossible to render a series expansion for one of the bodies [22]. He also expanded the mutual gravitational potential and the mutual gravitational torque of two bodies of arbitrary shape to the fourth order [23]. Paul offered an expansion of mutual potential between two gravitating bodies of finite size in a power series of the relative coordinates of their centers of mass, and the method was given without assuming any approximation and was valid universally without loss of any generality [24]. Based on series expansion of surface integrals, Werner considered the potential between homogeneous polyhedral [25]. Schutz expressed the mutual gravitational potential of two bodies of arbitrary shape to fourth order, and derived the expressions for the gravitational torques in a differential form [26]. Maciejewski investigated gravity and gravity gradient torque expressed by mutual gravitational potential of two rigid