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Active subspace approach to reliability and safety assessments of small satellite separation

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ABSTRACT

Ever-increasing launch of small satellites demands an effective and efficient computer-aided analysis approach to shorten the ground test cycle and save the economic cost. However, the multiple influencing factors hamper the efficiency and accuracy of separation reliability assessment. In this study, a novel evaluation approach based on active subspace identification and response surface construction is established and verified. The formulation of small satellite separation is firstly derived, including equations of motion, separation and gravity forces, and quantity of interest. The active subspace reduces the dimension of uncertain inputs with minimum precision loss and a 4th degree multivariate polynomial regression (MPR) using cross validation is hand-coded for the propagation and error analysis. A common spring separation of small satellites is employed to demonstrate the accuracy and efficiency of the approach, which exhibits its potential use in widely existing needs of satellite separation analysis.

1. Introduction

The rise in launch and use of small satellites is booming in recent years. As a mission-critical event, satellite separation directly influences performance of the satellite and its high-precision devices, even success of the entire launch. Any mechanical interference between the separating bodies is likely to be catastrophic [1–3]. A recent case is the failure of two Galileo navigation satellites launched in August 2014 wherein faulty separation systems are suspected to be the main culprits. Ground separation test is certainly a good way to avoid such failures, but costs too much time and money for small satellite development. It is therefore preferable to study an effective theoretical approach for this kind of reliability and safety assessment.

However, an accurate prediction of separation performance is tough since multiple influencing factors with uncertainty exist, such as the motion of launch vehicle, separation mechanism performance, satellite installation, gravitational perturbation, etc [4–6]. Moreover, the satellite itself inevitably possesses center of gravity (CG) offsets, which imparts undesirable, as well as unacceptable, lateral body rates on the satellite. The dynamics of small satellite separation has received the attention of several investigators [7–12]. How to fulfill a precise separation of small satellites from launch vehicle is one of the most important issues to be solved in the aerospace area [7]. As the conventional Monte Carlo analysis is too costly when the dimensionality of uncertain inputs is larger than ten, researchers generally try some design of experiments (DOE) methods in the quantity of interest for reliability and safety assessment under a limited number of uncertainties. For instance, Singaravelu [8] employs a Taguchi method to identify the probability density function (PDF) of the interest, which is more efficient compared to the Monte Carlo method but the accuracy is limited and the curse of dimensionality still exists.

With respect to multiple influencing factors in satellite separation, this paper presents a systematic analysis approach for convincing assessments of separation reliability and safety. The formulations of the rigid body separation dynamics and applied forces are derived in Section 2. A common separation system using the helical compression spring mechanism is modeled. Section 3 details the proposed approach consisting of active subspace identification, response surface construction, and the computational process. In Section 4, a typical small satellite separation exemplifies the approach and demonstrates its efficacy. Some conclusions and recommendations for future research are drawn in Section 5.

2. Problem modeling

The mathematical modeling required for separation simulation and dynamic analysis is explained. Orbital control strategy of the satellite and launch platform is not considered here. As shown in Fig. 1, relevant coordinate systems are defined. The geocentric inertial (ECI) coordinate system is taken as the reference, the orbital coordinate

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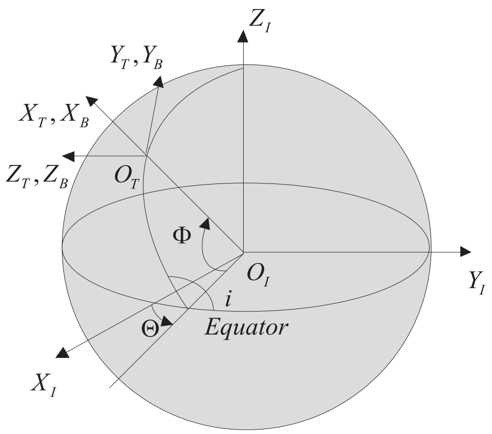

Fig. 1. Coordinate systems defined to describe the separation.

system as launch point inertial (LPI) system, and the satellite body inertial (SBI) coordinate system as the system to solve equations of motion.

2.1. Equations of motion

As shown in Fig. 2(a), the separation can be simplified as a dynamic process between two rigid bodies by employing Newton's second law and the momentum theorem. The body coordinate axes are defined as the principal of inertia and the product of inertia can be ignored [13,14]. The equations of motion for the six degrees of freedom (6DOF) in the respective body frame are

$$\frac{d}{dt}\begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \begin{Bmatrix} \omega_z v_y - \omega_y v_z \\ \omega_x v_z - \omega_z v_x \\ \omega_y v_x - \omega_x v_y \end{Bmatrix} + \frac{1}{m}\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix}, \tag{1}$$

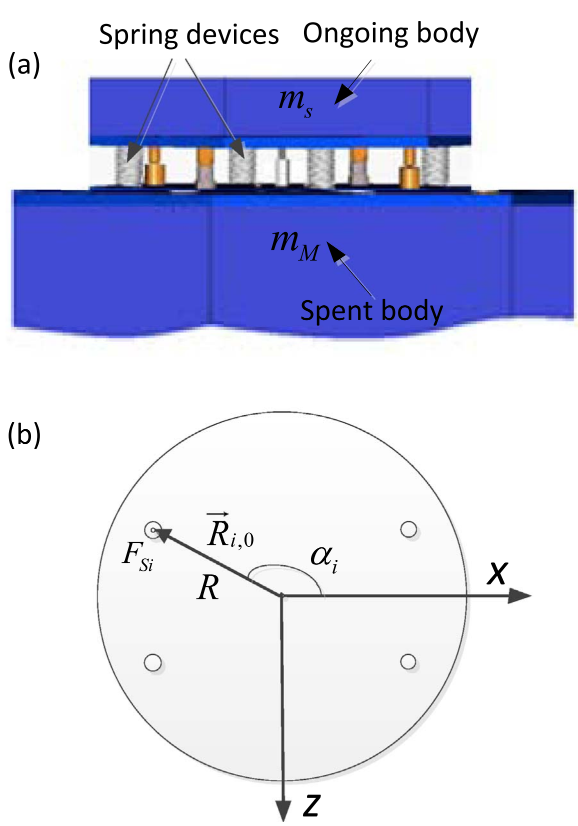

Fig. 2. (a) Schematic diagram of common satellite separation using spring devices; (b) Positions of spring forces in the separation plane.

$$\frac{d}{dt}\begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = [I]^{-1}\begin{Bmatrix} M_x - (I_z - I_y)\omega_y\omega_z \\ M_y - (I_x - I_z)\omega_x\omega_z \\ M_z - (I_y - I_x)\omega_x\omega_y \end{Bmatrix}. \tag{2}$$

where m is the mass of the body, $[I] = diag\,(I_x, I_y, I_z$ is the principal moment of inertia, $\overrightarrow{F_{ext}} = F_x\,\overrightarrow{i_b} + F_y\overrightarrow{j_b} + F_z\overrightarrow{k_b}$ is the total external force vector, and $\overrightarrow{M_{ext}} = M_x\,\overrightarrow{i_b} + M_y\overrightarrow{j_b} + M_z\overrightarrow{k_b}$ is the applied moment vector on satellite. $\vec{v} = v_x\,\overrightarrow{i_b} + v_y\overrightarrow{j_b} + v_z\overrightarrow{k_b}$ is the separation velocity and $\vec{\omega} = \omega_x\,\overrightarrow{i_b} + \omega_y\overrightarrow{j_b} + \omega_z\overrightarrow{k_b}$ is the angular velocity, including the components of yaw, pitch, and roll, respectively.

The kinematic differential equations, showing the relationship between angular velocity and Euler angles, are derived as

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\varphi & \sin\varphi\cos\theta \\ 0 & -\sin\varphi & \cos\varphi\cos\theta \end{bmatrix}\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}. \tag{3}$$

where ψ, θ, and φ are the attitude angles of yaw, pitch and roll, respectively.

Eqs. (1)–(3) are the governing equations of motion for the individual body undergoing separation and are nonlinear ordinary differential equations, which can be solved numerically after given initial conditions. If the satellite is installed and launched along different directions, corresponding changes of initial attitude angles can be specified adaptively. The equations therefore have good applicability.

2.2. Separation force and moment

The dynamic characteristics of separating bodies are modeled here under the influence of separation forces and moments. Separation force often comes from pre-compressed springs. As shown in Fig. 2(b), the distribution of these springs is usually designed to satisfy the general symmetry, though the body may have some offsets of CG. The force location is given as follows:

$$\alpha_i + \alpha_{N-i+1} = 2\pi \quad i = 1, 2, \ldots, N, \tag{4}$$

$$\overrightarrow{R_{i0}} = R\begin{bmatrix} \cos(\alpha_i) \\ 0 \\ \sin(\alpha_i) \end{bmatrix}, \; i = 1, 2, \ldots, N, \tag{5}$$

$$\Delta\overrightarrow{R_i} = \overrightarrow{R}_i^{\,2} - \overrightarrow{R}_i^{\,1} = ([T_{LB}]^{02} - [T_{LB}]^{01})\,\overrightarrow{R_{i,0}}, \tag{6}$$

where the superscript 1 stands for launch platform and the number 2 is for the satellite. $[T_{LB}]$ is the transformation matrix from LPI to the current SBI, described as Eq. (7). The separation force and moment vectors therefore can be obtained.

$$[T_{LB}] = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{bmatrix}, \tag{7}$$

$$\overrightarrow{F_{Si}} = = \begin{cases} K_i(\xi_i - \Delta R_i)\frac{\Delta\overrightarrow{R_i}}{\Delta R_i} & 0 \le \Delta R_i < L \\ 0 & \Delta R_i \ge L \end{cases}, \tag{8}$$

$$\overrightarrow{F_S} = \sum_{i=1}^{N}\overrightarrow{F_{Si}}, \tag{9}$$

$$\overrightarrow{M_S} = \sum_{i=1}^{N}(\overrightarrow{r_{Sj}} \times \overrightarrow{F_{Si}}), \tag{10}$$

where K_i, ξ_i and L represent the stiffness, initial deformation, and stroke length of the ith spring, respectively; $\overrightarrow{r_{Si}} = X_{Si}\,\vec{i} + Y_{Si}\vec{j} + Z_{Si}\vec{k}$ is