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Natural periodic orbit-attitude behaviors for rigid bodies in three-body periodic orbits ☆

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ABSTRACT

Trajectory design increasingly leverages multi-body dynamical structures that are based on an understanding of various types of orbits in the Circular Restricted Three-Body Problem (CR3BP). Given the more complex dynamical environment, mission applications may also benefit from deeper insight into the attitude motion. In this investigation, the attitude dynamics are coupled with the trajectories in the CR3BP. In a highly sensitive dynamical model, such as the orbit-attitude CR3BP, periodic solutions allow delineation of the fundamental dynamical structures. Periodic solutions are also a subset of motions that are bounded over an infinite time-span (assuming no perturbing factors), without the necessity to integrate over an infinite time interval. Euler equations of motion and quaternion kinematics describe the rotational behavior of the spacecraft, whereas the translation of the center of mass is modeled in the CR3BP equations. A multiple shooting and continuation procedure is employed to target orbit-attitude periodic solutions in this model. Application of Floquet theory and Poincaré mappings to identify initial guesses for the targeting algorithm are described. In the Earth–Moon system, representative scenarios are explored for axisymmetric vehicles with various inertia characteristics, assuming that the vehicles move along L_1/L_2 Lyapunov orbits as well as distant retrograde orbits. A rich structure of possible periodic behaviors appears to pervade the solution space in the coupled problem. The stability analysis of the attitude dynamics for the available families is included. Among the computed solutions, marginally stable and slowly diverging rotational behaviors exist and may offer interesting mission applications.

1. Introduction

Advances in orbital mechanics have demonstrated novel mission applications that exploit multi-body dynamical structures. The variety of potential applications range from astrophysical observatories, to solar sails, and to redirected natural bodies. When the attitude dynamics is coupled to a multi-body orbital regime, the spacecraft may also manifest complex rotational behaviors. Within the set of chaotic responses that is typical in a multi-body system, basic fundamental dynamical structures are also apparent and may aid in mission design when the attitude dynamics are incorporated. Periodic or quasi-periodic structures may potentially support ACS (Attitude Control System) operational modes for continuous data acquisition or communications, with coarse pointing requirements. A subset of the center subspace might be employed for safe-mode or long-term configurations. For example, an asteroid or a space station placed in a marginally stable subspace associated with the attitude modes is more likely to avoid tumbling in the long-term. Finally, manifold structures may guide large attitude slews.

In trajectory design, the Circular Restricted Three-Body Problem (CR3BP) is an approximation for the actual multi-body system, yet, its use is pivotal to grasp and employ the fundamental structures underlying the more complex dynamics. Thus, to explore the fundamental coupled behaviors, it is important to first understand the attitude dynamics when it is coupled to the CR3BP orbital regime. The earliest investigations from Kane, Marsh and Robinson consider the attitude stability of different satellite configurations, assuming that the spacecraft is artificially maintained precisely at the equilibrium points [1,2]. Successive studies introduce Euler parameters, i.e., quaternions, and Poincaré maps to explore the dynamics of a single body, one that remains fixed at the Lagrangian points [3,4]. The effects of the gravity torque along libration point orbits are examined by Wong, Patil and Misra for a single rigid vehicle in the Sun–Earth system [5]. Wong, Patil and Misra select Lyapunov and halo orbits for their investigation, and assume reference trajectories that are expressed in linear form; consequently, the results are acknowledged to apply to relatively small orbits close to the equilibrium points. Incorporating another simplification of the CR3BP, i.e., the Hill problem, Sanjurjo-Rivo et al. numerically reproduce the orbit-attitude coupled dynamics of a large dumbell satellite on halo and vertical orbits in

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the Earth–Moon system [6]. The application of Hill equations is limited to the vicinity of the smallest body in the system, when such a primary has practically negligible mass compared to the other attractor. Assuming that the spacecraft is in fast rotation, the attitude dynamics can be decoupled from the orbital dynamics by averaging the equations of motion over the "fast" angle [6]. Under this condition, it is demonstrated that incorporating sufficiently elongated structures may impact the stability of halo and L_2 vertical orbits in the Hill problem [7]. Later, Guzzetti et al. numerically investigate the coupled orbit and attitude equations of motion using the Lyapunov family as reference orbits and without the simplifications of the CR3BP nonlinear dynamics, but the rotation of the vehicle is limited to the orbital plane [8,9]. Guzzetti et al. also incorporate solar radiation pressure and simple flexible bodies in the investigation. The full three-dimensional coupled motion is explored by Knutson and Howell for a spacecraft comprised of multiple bodies in nonlinear Lyapunov and halo reference orbits [10,11]. Both Knutson and Guzzetti dedicate significant effort to identify conditions that determine bounded attitude solutions relative to the rotating frame in the CR3BP along nonlinear reference trajectories. Attitude maps are proven useful to recognize the orbital characteristics and the body inertia properties that enable the spacecraft to maintain its initial orientation with respect the rotating frame [12,13]. Most recently, Meng, Hao and Chen analyze the case of a dual-spin satellite in various halo orbits and, employing a semi-analytical expansion of the gravity torque, identify the main frequency components of the subsequent motion [14].

Along with stability diagrams based at the equilibrium points, mapping techniques as well as frequency analyses, investigation of periodic solutions may contribute to an understanding of the attitude dynamics when it is coupled with the CR3BP. In this investigation, solutions are sought that are simultaneously periodic in both the orbital and attitude states, when viewed in the rotating frame in the CR3BP. As is generally true for trajectories in the CR3BP, such orbit-attitude coupled solutions are expected to transition to higher-fidelity models with various degrees of success. A first example of such orbit-attitude periodic motion in the CR3BP is presented in [15]. The solution in [15] is limited to a disk-like spacecraft moving along L_1 Lyapunov orbits in the Earth–Moon system. Such behavior is also identified as a specific bifurcation from a reference elementary motion. The bifurcating solution is then corrected to render a precisely periodic dynamical solution using a single shooting algorithm. The work in [15] is evolved into a more systematic approach to identify orbit-attitude periodic behaviors in the CR3BP. In this investigation, a multiple-shooting formulation is described, one that is more generally applicable to the computation of complex solutions. Along with an improved numerical algorithm, the exploration of alternative reference trajectories, e.g., L_2 Lyapunov and distant retrograde orbits, and spacecraft topologies is included. An additional technique for the identification of periodic responses, i.e., Poincaré mapping, is also applied to the coupled orbit-attitude problem. Finally, the challenges in attempting to recognize ordered and predictable behaviors in higher-fidelity models is acknowledged. Yet, orbit-attitude periodic solutions from a simplified coupled model may be the stepping stone to identify and leverage potential natural motion in the actual – more dynamically complex – operational environment.

2. Dynamical model

Consider a single rigid spacecraft in the gravitational field emanating from two massive bodies P_1 and P_2. Assume that the bodies P_1 and P_2 are moving on circular orbits about their common barycenter, and their motion is unaffected by the presence of the spacecraft (whose mass is negligible compared to the masses of P_1 and P_2). The translational motion of the space vehicle is conveniently described by the Cartesian position coordinates (x, y, z) of the spacecraft center of mass relative to the barycenter of the system, as measured in a frame that rotates at the planetary system angular rate Ω. At time $t=0$, the rotating frame $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$, is aligned to the inertial frame $\hat{\boldsymbol{X}}$, $\hat{\boldsymbol{Y}}$, $\hat{\boldsymbol{Z}}$. At successive instants of time, the rotating frame is defined such that P_1 and P_2 remain on the $\hat{\boldsymbol{x}}$-axis and $\hat{\boldsymbol{z}}$ is equal to the normal vector $\hat{\boldsymbol{Z}}$ of the planetary orbiting plane, as depicted in Fig. 1. Referring to the figure, the body frame to describe the spacecraft orientation is also depicted, rendered by the tern of unit vectors $\hat{\boldsymbol{b}}_1$, $\hat{\boldsymbol{b}}_2$, $\hat{\boldsymbol{b}}_3$. In defining kinematical quantities, the notation ${}^a\bullet^c$ indicates that the motion of a generic c frame is observed from a generic a frame. For convenience, i denotes the inertial frame, r the rotating frame and b the body frame.

The system equations are normalized, such that the total mass of the system, the distance between the two attractors, the universal gravitational constant and the angular frequency Ω are unitary. The normalized period of P_1 and P_2 in their orbits about their barycenter is equal to 2π. After the normalization, the planetary system is dynamically represented by the mass parameter μ only, which is defined as the ratio between the mass of P_2 and the total mass of the system, neglecting the mass of the spacecraft (e.g., $\mu \approx 1.215 \times 10^{-2}$ for the Earth–Moon system). Assuming the mass of P_1 is greater than mass of P_2, the location of P_1 along the $\hat{\boldsymbol{x}}$-axis in nondimensional units is $-\mu$, whereas P_2 is located at $1-\mu$ nondimensional units from the barycenter.

To reproduce the orbital dynamics of the spacecraft, the gravity force is modeled neglecting the finite extension of the vehicle. Accordingly, the orbital behavior of the vehicle is equivalent to the response of a point-mass located at the center of mass of the vehicle. Perturbations that are equally significant when compared to the actual mass distribution, such as the solar radiation pressure, are also neglected in this simplified analysis. The resulting problem is familiar as the Circular Restricted Three-Body Problem (CR3BP), which is encapsulated in the following normalized scalar equations:

$$\boldsymbol{f}_x = \begin{cases} \dot{x} = v_x, \quad \dot{y} = v_y, \quad \dot{z} = v_z \\ \dot{v}_x = x + 2v_y - \dfrac{(1-\mu)(x+\mu)}{d^3} - \dfrac{\mu(x-1+\mu)}{r^3} \\ \dot{v}_y = y - 2v_x - \dfrac{(1-\mu)y}{d^3} - \dfrac{\mu y}{r^3} \\ \dot{v}_z = -\dfrac{(1-\mu)z}{d^3} - \dfrac{\mu z}{r^3}, \end{cases} \tag{1}$$

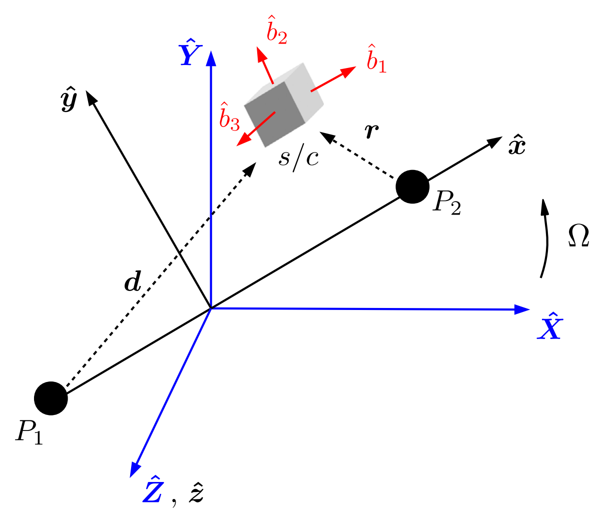

Fig. 1. Frame representations in the coupled orbit-attitude CR3BP. The blue vectors indicate the inertial frame, the black vectors define the CR3BP rotating frame, the red vectors represent the body frame. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

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