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Electric sail elliptic displaced orbits with advanced thrust model

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ABSTRACT

This paper analyzes the performance of an Electric Solar Wind Sail for generating and maintaining an elliptic, heliocentric, displaced non-Keplerian orbit. In this sense, this paper extends and completes recent studies regarding the performances of an Electric Solar Wind Sail that covers a circular, heliocentric, displaced orbit of given characteristics. The paper presents the general equations that describe the elliptic orbit maintenance in terms of both spacecraft attitude and performance requirements, when a refined thrust model (recently proposed for the preliminary mission design) is taken into account. In particular, the paper also discusses some practical applications on particular mission scenarios in which an analytic solution of the governing equations has been found.

1. Introduction

The Electric Solar Wind Sail (E-sail) is a recent, propellantless, propulsion system concept [1–3] that consists of a number of thin, long, and conducting tethers, which are deployed and stretched out using the centrifugal force obtained by spinning the spacecraft around its symmetry axis [4]. The tethers, which are kept at a high positive potential by an onboard electron gun, experience Coulomb drag [5] by interacting with the solar wind plasma stream and generate a propulsive thrust [6,7]. The E-sail is theoretically able to allow a spacecraft to reach and maintain highly non-Keplerian orbits, which are difficult, or even impossible, to be generated with more conventional propulsion systems such as chemical or electrical thrusters [8]. Within this set of highly non-Keplerian closed trajectories, a class of great practical importance is that of displaced orbit, whose orbital plane does not contain the primary body.

The concept of displaced non-Keplerian orbit (DNKO) has received a great interest in the recent scientific literature [9,10]. Most of the available studies involve the use of a photonic solar sail as the propulsion system for orbital maintenance. An excellent survey of the potential applications for DNKOs can be found in the comprehensive survey by McKay et al. [11].

This paper, instead, focuses on DNKOs maintained by an E-sail. A preliminary analysis of such a subject has been given in Ref. [12]. However, that study [12] was based on a simplified mathematical model of the E-sail performance, characterized by two main assumptions. On one side the thrust modulus variation with the Sun-spacecraft distance r was chosen proportional to $1/r^{7/6}$, whereas more recent plasmadynamic simulations [13] have shown that the thrust modulus scales as the inverse of the Sun-spacecraft distance, i.e. as $1/r$. The second simplified assumption used in Ref. [12] was that the propulsive acceleration was assumed to be independent of the spacecraft attitude and the thrust direction formed an angle (the cone angle) approximatively equal to one half of the angle between the Sun-spacecraft line and the normal to the sail plane (the pitch angle). In a recent paper Yamaguchi and Yamakawa [14] have proposed a more accurate mathematical model in which the thrust modulus and direction are both parameterized as functions of the E-sail attitude. The refined thrust model has been used in Ref. [15] to analyze the E-sail performances in an asteroid deflection mission, and in Ref. [16] for studying circular DNKOs in an E-sail-based interplanetary scenario.

The aim of this paper is to extend the results of Ref. [16] to the more general (and complex) case of elliptic heliocentric DNKOs using, again, the refined thrust model by Yamaguchi and Yamakawa [14]. In fact, an interesting potential application of an elliptic DNKO is the observation of the polar regions of a planet or, in general, of a celestial body. The paper starts by looking for the values of thrust modulus and direction necessary to maintain a generic, elliptic, DNKO. In this context, the thrust modulus variation along the orbit, due to the time variation of the spacecraft-focus distance, can be obtained by suitably adjusting the E-sail tether voltage [4,17].

Some special mission scenarios are discussed, in particular the case of a planet following displaced orbit in which the E-sail-based spacecraft moves along the displaced orbit with the same angular velocity of a reference planet. In this case, the DNKO eccentricity is assumed to be equal to that of the planetary orbit, but the semimajor axes of the two

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Nomenclature

$\boldsymbol{a}$	propulsive acceleration, [mm/s^2]
a_c	spacecraft characteristic acceleration, [mm/s^2]
a	planet's orbit semimajor axis, [au]
a_ρ, a_z	components of the propulsive acceleration, [mm/s^2]
d	planet-spacecraft distance, [au]
e	planet's orbit eccentricity
H	vertical displacement of non-Keplerian orbit, [au]
h_z	vertical component of the spacecraft specific angular momentum, [au^2/TU]
$\hat{\boldsymbol{n}}$	unit vector normal to the E-sail mean plane
O	Sun's center-of-mass
O'	projection of Sun's center-of-mass on the DNKO plane
P	perihelion of the planet's orbit
P'	pericenter of the displaced orbit
q	ratio of spacecraft radial coordinate to planet's heliocentric distance
$\boldsymbol{r}$	spacecraft position vector (with $r \triangleq \lVert \boldsymbol{r} \rVert$), [au]
r_p	planet's heliocentric distance, [au]
$r_\oplus$	reference distance (1 au)
S	spacecraft center-of-mass
z	spacecraft vertical coordinate, [au]
α	cone angle, [deg]
α_n	pitch angle, [deg]
γ	dimensionless propulsive acceleration
$\mu_\odot$	Sun's gravitational parameter, [km^3/s^2]
ν	planet's true anomaly, [deg]
ω	angle between spacecraft and planet apse line, [deg]
ψ	elevation angle, [deg]
ρ	spacecraft radial coordinate, [au]
τ	voltage parameter
θ	spacecraft angular coordinate, [deg]Subscripts
max	maximum
min	minimumSuperscripts
	time derivative
$\wedge$	unit vector

orbits can be different in order to obtain a parametric study of this scenario. Indeed, the ratio of the two semimajor axes and the amount of displacement (with respect to the planet's orbital plane) are two fundamental parameters that must be carefully chosen in order to obtain a feasible DNKO.

The paper is organized as follows. The next section presents the mathematical model that has been used for the analysis of the spacecraft performances in a (general) heliocentric, elliptic, displaced orbit. Section 3 introduces the concept of a particular displaced heliocentric orbit and illustrates, through numerical simulations, the E-sail capabilities in some interesting mission scenario. Finally, the Conclusion section presents the outcome of this work.

2. Mathematical model

Consider a planet's heliocentric orbit as a reference (Keplerian) orbit and introduce a classical, perifocal, reference frame $\mathcal{T}(O;\, x,\ y,\ z)$, see Fig. 1.

The origin of $\mathcal{T}$ is at the Sun's center-of-mass O, the plane (x,y) coincides with the orbital plane (the x-axis points towards the celestial body's perihelion P), and the z-axis is orthogonal to the orbital plane in the direction of the planet's angular momentum vector. The planet's position along the orbit is given by its true anomaly ν, measured counterclockwise from the x-axis. Assume that the DNKO orbital plane is parallel to the (x,y) plane, and is placed at a (given) distance H with respect to the Sun's center of mass O. Moreover, the DNKO primary focus O' is assumed to coincide with the orthogonal projection of O onto the DNKO plane, whereas P' is the DNKO perihelion where the O'-spacecraft distance is minimum, see Fig. 1.

The spacecraft position S along the DNKO orbit is conveniently described by a cylindrical coordinate system $\mathcal{T}_c(O';\, \rho,\ \theta,\ z)$, where ρ coincides with the O'-S distance, and θ is the spacecraft angular coordinate measured counterclockwise starting from the O'-P' direction. Note that, by definition, ν and θ differ for a constant angle ω that coincides with the angle between the x-axis and the O'-P' direction, see Fig. 1.

To maintain an elliptic DNKO, the spacecraft needs to be equipped with a propulsion system capable of providing a continuous and, within some limits, adjustable thrust. To that aim, in principle the spacecraft could use a propulsion system that exploits some kind of propellant to generate the required continuous thrust, such as a classical electric thruster. However, if the spacecraft is planned to track a DNKO for a long time interval, on the order of several months or years, the only feasible option is to resort to a propellantless propulsion system, which removes the constraint involving the finite amount of available propellant. Taking into account the current technology level, the natural choice is confined to either a photonic solar sail, or to the more recent E-sail. Unlike the former, the latter enables a simpler thrust modulation to be obtained by adjusting the tether voltage.

Accordingly, this paper assumes that the spacecraft is equipped with an E-sail whose more recent thrust model has been introduced by Yamaguchi and Yamakawa [14] and thoroughly analyzed, for a minimum-time heliocentric-transfer mission scenario, in a recent paper by Quarta and Mengali [18]. To summarize the advanced thrust model [14] conveniently, let $\hat{\boldsymbol{n}}$ be the unit vector normal to the nominal plane containing the E-sail tethers, in the direction opposite to the Sun. According to Refs. [14,18], the E-sail propulsive acceleration vector $\boldsymbol{a}$ is a suitable function of both the Sun-spacecraft distance r, and the sail pitch angle $\alpha_n \in [0,\ 90]°$, defined as the angle between the direction of $\hat{\boldsymbol{n}}$ and the direction of the spacecraft position unit vector $\hat{\boldsymbol{r}}$ (i.e. the direction of the spacecraft position vector measured from O), viz.

$$\boldsymbol{a} = \tau\, a_c \left(\frac{r_\oplus}{r}\right) \gamma\, \hat{\boldsymbol{a}} \tag{1}$$

where a_c is the spacecraft characteristic acceleration, defined as the maximum modulus of $\boldsymbol{a}$ at a Sun-spacecraft reference distance $r_\oplus \triangleq 1$ au, τ is a dimensionless (voltage) parameter that models the thrust modulation by suitably adjusting the tether voltage, and

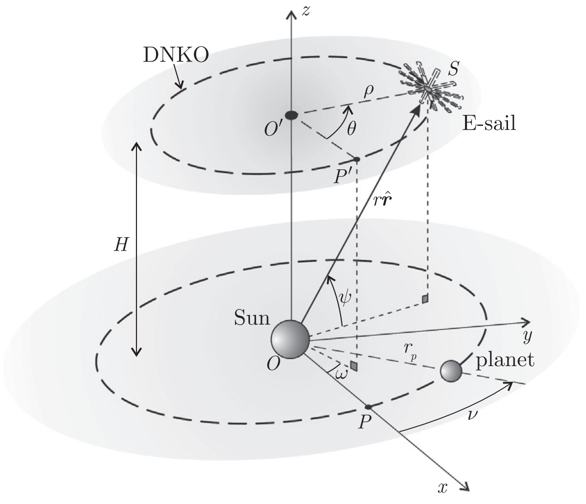

Fig. 1. Reference frame and DNKO schematic view.