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Optimal thrust level for orbit insertion

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ABSTRACT

The minimum-fuel orbital transfer is analyzed in the case of a launcher upper stage using a constantly thrusting engine. The thrust level is assumed to be constant and its value is optimized together with the thrust direction. A closed-loop solution for the thrust direction is derived from the extremal analysis for a planar orbital transfer. The optimal control problem reduces to two unknowns, namely the thrust level and the final time. Guessing and propagating the costates is no longer necessary and the optimal trajectory is easily found from a rough initialization. On the other hand the initial costates are assessed analytically from the initial conditions and they can be used as initial guess for transfers at different thrust levels. The method is exemplified on a launcher upper stage targeting a geostationary transfer orbit.

1. Introduction

Mass minimization is a major concern for the design of launch vehicles. The fuel required to reach the targeted orbit depends on both the thrust level and the thrust orientation along the trajectory. Finding the minimum-fuel trajectory is an optimal control problem that can be addressed by the Pontryaguin Maximum Principle (PMP). Due to its utmost importance, this problem has received a considerable attention from academics and industrials since the beginning of the space age [1–4] and it is still an active topic of research. We can distinguish the impulsive formulation [5–9] which assumes instantaneous velocity changes and the continuous formulation which accounts for the engine limited thrust level [10,11]. Depending on the available acceleration level, we can also separate the high-thrust and the low-thrust case which requires specific solution methods.

In a few cases, when the dynamical model is sufficiently simple, an analytical solution may be derived from the PMP necessary conditions. Among such well-known solutions for minimum-fuel trajectory problems, we can mention the following ones.

- The Goddard's problem [12] for a vertical launched rocket.
- The flat Earth model with constant gravity and constant acceleration [1,13]
- The Edelbaum's model for a low thrust transfer between circular orbits [2]

The flat Earth model and the constant acceleration model do not have the sufficient representativeness to correctly assess the launcher optimal performance [14]. The problem must be formulated in a central gravity field and considering the actual engine thrust level. Depending on the launcher and the mission specifications, the thrust level may be either prescribed (in that case the problem is equivalent to a minimum-time problem) or freely optimized between some bounds [15]. The thrust direction may also be restricted due to path constraints.

Efficient approaches have been developed for the endo-atmospheric leg of an ascent trajectory accounting for stringent aerodynamics constraints, and closed-loop command laws have been derived compliant with an on-board guidance [16–18]. For the exo-atmospheric flight closed-loop solutions relying on a linear gravity approximation prove accurate enough regarding the terminal orbit constraints [19]. On the other hand there exist no analytical solution to the orbit transfer problem when considering the central inverse square gravity field. Numerical procedures must be used that are classified between direct and indirect methods [20,21]. Direct methods [22–29] discretize the optimal control problem in order to rewrite it as a nonlinear large scale optimization problem. Numerous efficient software packages such as IPOPT, BOCOP, GESOP, SNOPT, WORHP ..., are available making the method suitable to a wide range of applications, particularly for low-thrust transfer optimization [24,28]. These methods are nevertheless computationally demanding and they may be not very accurate. On the other hand indirect methods are based on the Pontryagin Maximum Principle (PMP) [30,31] which reduces the problem to a system of nonlinear equations. Applying the PMP to the minimum-fuel problem yields in the regular case the optimal thrust direction aligned with the velocity costate, whereas the thrust level (if optimized) is driven by the

Abbreviations: OCP, Optimal Control Problem; PMP, Pontryaguin Maximum Principle; TPBVP, Two Point Boundary Value Problem; NLP, Non Linear Programming; GTO, Geostationary Transfer Orbit

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sign of a switching function. Such switches induce additional issues : their number is a priori unknown and they must be detected accurately within the simulation process in order to keep a good differentiability of the problem [32]. Furthermore there may exist singular arcs which require further theoretical analysis and specific solution methods [33,34]. When properly initialized shooting methods are very fast and produce high accuracy solutions. The main issue lies in their sensitivity to the initial costate guess. Even for the minimum time problem [35,36] when only the thrust direction is optimized (regular case, prescribed thrust level), finding the unknown initial costate proves numerically challenging, and it may discourage from using an indirect method.

Various approaches can be envisioned to build a satisfying initial costate guess and benefit from the efficiency of the indirect method. In [37] the impulse transfer solution is used to provide a good initial guess to the shooting algorithm. This method is based on the fact that a continuous high-thrust orbit transfer shares similarities with the impulse transfer as outlined in [5,38]. Multiple shooting reduces the overall sensitivity by splitting the trajectory in several arcs at the expense of additional unknowns and boundary conditions. In [39] a multiple shooting method parameterized by the number of thrust arcs is used to solve an Earth-Mars transfer. Homotopic approaches [40] solve a series of optimization problems by continuous transformation starting from a known solution. In [35,41,42] a differential continuation method linking the minimization of the L^2-norm of the control to the minimization of the consumption is used to solve the low-thrust orbit transfer around the Earth. In [43] simplified formulas are established by interpolating many numerical experiments, which allows a successful initialization for the minimal time orbit transfer problem, in a given range of nearly circular initial and final orbits. Based on that initial guess and on averaging techniques, the authors of [44] implement in the software T3D continuation and smoothing processes in order to solve minimal time or minimal fuel consumption orbit transfer problems. Particle swarm [45] or genetic algorithms [46] can also be used to explore largely the variables space and produce a satisfying initial solution. We can also mention mixed methods that use a discretization of the PMP necessary conditions and then apply a large-scale equation solver [47] and dynamic programming methods that search for the global optimum in a discretized state space by solving the Hamilton-Jacobi-Bellman equation [48].

This paper addresses the minimum-fuel orbital transfer in the particular case of an engine constantly thrusting at the same thrust level. Although this assumption may seem restrictive, this case is of great practical importance since rocket engines are generally designed for a reference thrust level. The targeted application is the flight of a high thrust launcher upper stage. The initial conditions are prescribed resulting from the previous stage flight. The final conditions are defined in terms of orbital parameters.

When the engine constant thrust level is considered as a free optimization parameter (as is the case in preliminary design studies) an additional optimality condition has to be written with the PMP equations. This condition can be exploited in the planar case to derive a closed-loop solution for the thrust optimal direction. Guessing and propagating the costates is no longer necessary and the minimum-fuel trajectory problem is reduced to a nonlinear system of 2 equations (targeted apogee and perigee) with 2 unknowns (thrust level and final time). This problem is easily solved from a rough initial solution.

On the other hand the initial costates corresponding to the optimal trajectory are derived analytically from the initial conditions. These costates can be used as initial guess for instances of the same minimum-fuel problem when the thrust level is no longer a free parameter.

The text is organized as follows. In Section §2 the optimal control problem is formulated and the extremal conditions are analyzed. A closed-loop control law is derived in the planar case and the solution method is presented. In Section §3 the method is applied to a representative example of a launcher targeting a geostationary transfer orbit. A sensitivity analysis on the thrust level illustrates how the analytical costates can be used as starting point to solve the minimum-fuel problem at non-optimal thrust level. The extension to low-thrust transfers is also discussed.

2. Problem formulation and analysis

This section formulates the Optimal Control Problem (OCP) under consideration. The problem is analyzed by applying the Pontryaguin Maximum Principle (PMP) and a closed-loop control is derived from the first order necessary conditions in the planar case.

2.1. Optimal control problem

The problem consists in finding the minimal-fuel trajectory to go from given injection conditions to a targeted orbit with a constantly thrusting engine. The Earth is modeled as a sphere, with its center at the origin of an inertial frame. The vehicle is considered as a material weighting point with position $\vec{r}(t)$, velocity $\vec{v}(t)$, mass m(t) submitted to the Earth acceleration gravity denoted $\vec{g}(\vec{r})$and to the engine thrust. The thrust level T is constant with a burned propellant exhaust velocity equal to v_e. The engine is ignited at the initial date t_0 and it cannot be turned off before the orbit insertion at t_f. The thrust direction can be chosen freely and it is orientated along the unit vector $\vec{u}(t)$.

Applying the fundamental dynamics principle in the Earth-centered inertial frame yields the motion equations.

$$\begin{cases} \dot{\vec{r}} &= \vec{v} \\ \dot{\vec{v}} &= \vec{g} + \frac{T}{m}\vec{u} \\ \dot{m} &= -\frac{T}{v_e} \end{cases} \tag{1}$$

The dependencies on time (for $\vec{r}$, $\vec{v}$, m, $\vec{u}$) and on position (for $\vec{g}$) have been omitted for conciseness. A planar transfer is considered and all vectors are of dimension 2.

In order to formulate an optimal control problem, we consider as state variables $\vec{r}(t)$,$\vec{v}(t)$ and m(t). The control variables are the thrust direction $\vec{u}(t)$, the thrust level T and the final time t_f. The thrust level T can take any finite positive value between a lower bound T_{min} and an upper bound T_{max}. We assume that these bounds are sufficiently large and that they are not active on the optimal solution.

The initial state at the engine ignition results from the ascent trajectory flown by the launcher lower stages. This initial state is completely prescribed. The final state is constrained by the targeted orbit defined by the apogee and perigee altitudes denoted respectively h_A and h_P. The apogee and perigee altitudes actually achieved at the final date are denoted respectively ψ_A and ψ_P and they depend on the final position $\vec{r}(t_f)$ and velocity $\vec{v}(t_f)$

The optimal control problem is formulated under the Mayer form with a final cost.

$$\min_{\vec{u}(t),T,t_f} J = -m(t_f) \quad s.\ t. \quad \begin{cases} \dot{\vec{r}} = \vec{v} \\ \dot{\vec{v}} = \vec{g} + \frac{T}{m}\vec{u} \\ \dot{m} = -\frac{T}{v_e} \end{cases} \quad with$$

$$\begin{cases} \vec{r}(t_0) = \vec{r}_0 \\ \vec{v}(t_0) = \vec{v}_0 \\ m(t_0) = m_0 \end{cases} \quad fixed\ initial\ state$$

$$\begin{cases} \psi_A[\vec{r}(t_f), \vec{v}(t_f)] &= h_A \\ \psi_P[\vec{r}(t_f), \vec{v}(t_f)] &= h_P \end{cases} \quad constrained\ final\ state \tag{2}$$