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Robustness analysis method for orbit control

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ABSTRACT

Satellite orbits require periodical maintenance due to the presence of perturbations. However, random errors caused by inaccurate orbit determination and thrust implementation may lead to failure of the orbit control strategy. Therefore, it is necessary to analyze the robustness of the orbit control methods. Feasible strategies which are tolerant to errors of a certain magnitude can be developed to perform reliable orbit control for the satellite. In this paper, first, the orbital dynamic model is formulated by Gauss' form of the planetary equation using the mean orbit elements; the atmospheric drag and the Earth's non-spherical perturbations are taken into consideration in this model. Second, an impulsive control strategy employing the differential correction algorithm is developed to maintain the satellite trajectory parameters in given ranges. Finally, the robustness of the impulsive control method is analyzed through Monte Carlo simulations while taking orbit determination error and thrust error into account.

1. Introduction

The trajectories of satellites deviate from their nominal orbits due to orbital perturbations, such as asymmetry of the gravitational field and atmospheric drag. Therefore, station-keeping is crucial to ensure that a spacecraft can operate in the vicinity of its nominal orbit. In the control system, however, inevitable random errors caused by the orbit determination process and the thrust implementation device may lead to failure of the entire spacecraft mission. Hence, it is necessary to design a station-keeping strategy that is tolerant to a certain magnitude of errors. Many studies have addressed this issue during the past few decades. A number of orbit control methods have been developed using impulsive maneuvers or continuous maneuvers to control the spacecraft formation flight [1–10]. Various types of errors have been considered in control algorithms to test their applicability in outer space. Howell and Barden [11] applied a station-keeping strategy to multiple spacecraft in formation near a libration point and Monte Carlo simulations were conducted with various errors. Tragesser and Skrehart [12] took account of the short period fluctuations of the orbit elements, caused by the oblateness perturbations, in the guidance scheme for low earth orbit (LEO) satellite and Monte Carlo simulations incorporating the navigational error and the control error were performed to access the performance of the guidance approach. Lian et al. [13] addressed a discrete-time sliding mode control for the station-keeping of real Earth-Moon libration point orbits and Monte Carlo results verified that the proposed approach was applicable in keeping the satellite within a close range of the nominal orbits. Bruijin and Gill [14] studied the impact of sensor and actuator error on formation control accuracy and propellant consumption with two impulsive control methods. Rozanov and Guelman [15] developed a variable structure closed-loop control to manage the effects of unpredictable atmosphere density for aerocapture, and Monte Carlo simulations were performed to test the robustness of the algorithm. Qi et al. [16] developed an impulsive control method based on two-level differential corrections to maintain the flight formation in the circular restricted three-body problem and to fully exploit the predetermined error bound. Thus, the robustness analysis is necessary to offer proposals to the design of orbit control law and to ensure the execution of space missions.

The primary goal of this work is to develop a method for evaluating the robustness of orbit control methods. An ocean satellite of China called HY-2, which is mainly used for the observation and research of the ocean environment, is taken as the chief object of this study. To meet the mission requirements, HY-2 operates in a near-circular sun-synchronous orbit. In Section 2, its equations of motion are described employing Gauss' form of the planetary equations, and the perturba-

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tions exerted on the satellite are considered. Note that the mean orbit elements are employed in this research in preparation for the orbit analysis. This section also presents the differential correction algorithm that is utilized to achieve a precise orbit maneuver. Section 3 demonstrates the designed impulsive station-keeping strategy, and its performance is verified through orbit simulation. In Section 4, random error models of position, velocity and thrust are introduced. Monte Carlo simulations with these random errors are conducted to analyze the robustness of the station-keeping strategy. Finally, Section 5 concludes the work.

2. Orbit dynamic model

The equations of motion of the spacecraft are formulated using the mean orbit elements along with external perturbation forces such as the atmospheric drag and the Earth's non-spherical perturbations.

2.1. Gauss' form of the planetary equations and perturbation forces

The real orbit of spacecraft, varying in time in outer space, can be considered to be a sequence of osculating Keplerian orbits arranged in chronological order. This notion forms the foundation of the method of variation of orbit elements, which yields a set of first-order differential equations of the variation of the osculating Keplerian orbit elements with time [17]. The set of Gauss' form of the planetary equations, applicable to non-conservative forces, is just one of the results from this idea. The set consists of six first-order differential equations describing the variations of the six orbit elements and is shown in Eq. (1).

$$\left\{\begin{array}{l} \frac{da}{dt} = \frac{2a^2}{h}(e\sin\theta f_r + \frac{p}{r}f_u) \\ \frac{de}{dt} = \frac{1}{h}(p\sin\theta f_r + ((p+r)\cos\theta + er)f_u) \\ \frac{di}{dt} = \frac{r\cos(\omega+\theta)}{h}f_h \\ \frac{d\Omega}{dt} = \frac{r\sin(\omega+\theta)}{h\sin i}f_h \\ \frac{d\omega}{dt} = -\frac{1}{he}\cos\theta p f_r + \frac{1}{he}(p+r)\sin\theta f_u - \frac{r}{h\sin i}\sin(\omega \\ \qquad + \theta)\cos i f_h \\ \frac{d\theta}{dt} = \frac{h}{r^2} + \frac{1}{he}(p\cos\theta f_r - (p+r)\sin\theta f_u) \end{array}\right\} \tag{1}$$

The semimajor axis, eccentricity, and inclination are denoted by a, e, and i, respectively. The right ascension of the ascending node is Ω, and the argument of perigee is ω. The true anomaly of the satellite is denoted by θ. The distance between the Earth and the satellite is r. The components of the perturbation force in the orbital frame are denoted by f_r, f_u, and f_h, in which f_r is along the orbit position vector, f_u is perpendicular to f_r in the orbital plane, pointing in the direction of motion, and f_h is orthogonal to these vectors using the right-hand rule. The semilatus rectum is denoted by p, the magnitude of the angular momentum is h, and the gravitational constant is μ.

The perturbation force in Eq. (1) includes the atmospheric drag and the Earth's non-spherical perturbations because the HY-2 satellite operates in LEO; forces exerted by the Earth and the atmosphere around the Earth are the dominate influences on the satellite orbit. According to aeronautics, atmospheric drag exerted on the satellite can be described through Eq. (2)

$$D = C_D\frac{1}{2}\rho v_a^2 S \tag{2}$$

where C_D denotes the drag coefficient, and ρ is the atmospheric density obtained from the U.S. Standard Atmosphere 1976 model (SA76) [18]. In this paper, ρ is considered to be a constant because the variation of atmosphere density is negligible when the satellite is in the vicinity of the nominal orbit. v_a is the velocity of the satellite relative to the Earth. The cross-sectional area of the satellite is denoted by S. To simplify Eq. (2), we assume that the atmosphere is spherically symmetric and static and that the windward area of the spacecraft does not change. Thus, the atmospheric drag produces only the tangential acceleration of the satellite in the velocity frame as is shown in Eq. (3) where f_n, f_t, and f_h are the components of the atmospheric drag in the velocity frame in which f_t points in the velocity direction, f_h is normal to the orbital plane, positive in the direction of the angular momentum vector, and f_n is orthogonal to these two vectors using the right-hand rule. v is the satellite velocity in the tangential direction of the orbit.

$$\left\{\begin{array}{l} f_n = 0 \\ f_t = -\frac{1}{2m}C_D S\rho v^2 \\ f_h = 0 \end{array}\right. \tag{3}$$

$$\sin\gamma = e\sqrt{\frac{\mu}{p}}\frac{\sin\theta}{v} \tag{4}$$

The perturbation force in Eq. (1) is described in the orbital frame, whereas the atmospheric drag in Eq. (3) is in the velocity frame. Therefore, a coordinate system transformation is required based on the normal direction of the orbital plane with a rotation angle γ when substituting the atmospheric drag into Eq. (1). The expression of γ is shown in Eq. (4).

The gravitational potential is a complex high order polynomial function in which the sectorial harmonics can be largely offset over many periods; we may therefore neglect the effect of these harmonics for many applications. Additionally, the gravitational potential is always truncated, retaining only the terms that contribute significantly to the orbit. Thus, only the accelerations produced by J_2, J_3, and J_4-term from the gravitational potential are included in this research, and the expression is presented in Eq. (5).

$$\begin{array}{l} \Delta g_x = \frac{\mu x}{r^3}\sum_{n=2}^{4} J_n(R_E/r)^n P'_{n+1}(\sin\varphi) \\ \Delta g_y = \frac{\mu y}{r^3}\sum_{n=2}^{4} J_n(R_E/r)^n P'_{n+1}(\sin\varphi) \\ \Delta g_z = \frac{\mu}{r^2}\sum_{n=2}^{4} J_n(R_E/r)^n (n+1)P_{n+1}(\sin\varphi) \end{array} \tag{5}$$

In Eq. (5), P_n and P'_n are the Legendre function of the first kind of degree n and their derivatives. x,y, and z are the coordinates of the spacecraft in the inertial frame. r and φ are defined by $r = \sqrt{x^2+y^2+z^2}$ and $\sin\varphi = z/r$, respectively. The radius of the Earth is denoted by R_E. A coordinate system conversion is also required when substituting Eq. (5) into Eq. (1); the method is similar to the coordinate transfer of the atmospheric drag in Eq. (3) and therefore not shown.

In this study, the mean orbit elements are employed to explicitly depict the long-term variations of the orbit elements. The conversion from osculating orbit elements to mean orbit elements is omitted as it can be easily obtained from Ref. [19].

2.2. Differential correction algorithm

Suppose a satellite is initially at a point in space with a state vector $\boldsymbol{x}_0$. After a maneuver $\Delta\boldsymbol{V}$, it arrives at point $\boldsymbol{x}^t$ at $t = t_0 + \Delta t$. The state of the spacecraft at the desired point is denoted by $\boldsymbol{x}^d$. Therefore, the elimination of the difference between $\boldsymbol{x}^t$ and $\boldsymbol{x}^d$ requires a suitable $\Delta\boldsymbol{V}$ obtained from numerical methods. The satellite would then arrive in the vicinity of the desired place within a preset precision [20]. In this paper, the differential correction algorithm (DCA) with the advantage of fast convergence is utilized to find the appropriate $\Delta\boldsymbol{V}$.

For HY-2, of the six orbit elements, a, e, and ω require correction during the rotation around the Earth. The reason that only these three elements need to be controlled will be explained in Section 3. Thus in this paper, the free variable vector $\boldsymbol{X}$ is

$$\boldsymbol{X} = \begin{bmatrix} a \\ e \\ \omega \end{bmatrix} \tag{6}$$