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Reentry trajectory optimization with waypoint and no-fly zone constraints using multiphase convex programming

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ABSTRACT

This study proposes a multiphase convex programming approach for rapid reentry trajectory generation that satisfies path, waypoint and no-fly zone (NFZ) constraints on Common Aerial Vehicles (CAVs). Because the time when the vehicle reaches the waypoint is unknown, the trajectory of the vehicle is divided into several phases according to the prescribed waypoints, rendering a multiphase optimization problem with free final time. Due to the requirement of rapidity, the minimum flight time of each phase index is preferred over other indices in this research. The sequential linearization is used to approximate the nonlinear dynamics of the vehicle as well as the nonlinear concave path constraints on the heat rate, dynamic pressure, and normal load; meanwhile, the convexification techniques are proposed to relax the concave constraints on control variables. Next, the original multiphase optimization problem is reformulated as a standard second-order convex programming problem. Theoretical analysis is conducted to show that the original problem and the converted problem have the same solution. Numerical results are presented to demonstrate that the proposed approach is efficient and effective.

1. Introduction

Rapid trajectory optimization is still an open challenging field among many engineering fields including aerospace engineering. A typical problem is to efficiently plan a feasible trajectory for a hypersonic vehicle, which is constrained by heat rating, dynamic pressure, normal load, and other constraints related to the mission. To address such a highly constrained nonlinear dynamical optimization problem, a number of methods have been proposed. Generally, these methods can be classified into two categories: direct and indirect methods [1]. The indirect method, mainly based on Pontryagin's maximum principle, results in a Hamiltonian Boundary Value Problem (HBVP) via definition of the costate variables in addition to the state variables [2,3]. The direct methods usually convert the original infinite-dimensional dynamical optimization problem to a finite-dimensional nonlinear programming problem (NLP) through parameterization techniques, such as the widely used pseudospectral methods [4–7]. Next, the resulting NLP can be solved by sequential quadratic programming (SQP) [8]. Some software packages have been developed based on direct methods,such as GPOPS [6], GPOCS [9], etc.. However, these nonlinear programming methods cannot provide a priori guarantees on the convergent rate, furthermore, they cannot necessarily guarantee that the global optimal solution is obtained [10].

Because of the polynomial-time complexity and the unique theoretical advantages, convex optimization has been wildly used in the optimal control or programming for decades. Recently, this technique has been successfully implemented in aerospace guidance and trajectory planning, including planetary powered soft landing [11,12,10], rendezvous and proximity operations [13], control of swarms of spacecrafts [14], and entry trajectory optimization [15,16]. These outstanding innovative works demonstrate the great advantages of convex optimization over than other NLP solving methods: (1) rapid convergent rate; (2) insensitivity to the initial guess. Note that most aerospace engineering problems are non-convex in nature; hence, the works aforementioned mainly focus on the relaxation of the concave constraints and thus an equivalently relaxed convex optimization problem, or an approximately equivalent second-order conic problem (SOCP); as a result, the efficient primal-dual interior-point methods [17,18] will be used to obtain the optimal solution.

This work mainly reconsidered the CAV reentry trajectory optimization problem [19] with waypoint and NFZ constraints by using the recently emerged convex programming method. For the entry trajectory optimization under convex optimization framework, the authors of [15,16] have produced great innovative works: given the profile of the

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angle of attack, the corresponding SOCP is setup by using successive approximation and convexification techniques, and then the optimal trajectory is obtained by a commercial SOCP solver MOSEK [18]. During this procedure, the non-convex path constraints, including heat flux, dynamic pressure and normal load, are numerically converted to the inequality constraints regarding the vehicle altitude, and the non-convex NFZ constraint indices are approximated by a series of linear convex inequalities. The numerical results for trajectory optimization of CAV and X-33 show that the sequential convex programming method proposed in [15] is very efficient. Note that in these works, a velocity-dependent angle-of-attack (AOA) profile is assumed to be a priori specified. Such an assumption simplifies the problem because of the reduction of the number of control variables; however, the AOA profile should be obtained by other methods and is not necessarily optimal.

In this paper, we develop a new convex programming method to simultaneously optimize two control variables, namely the normalized coefficient of lift and the bank angle, in the time domain on the basis of the method proposed in [15], in which the motion of vehicle is defined with respect to energy. In addition to the constraints on heat flux, dynamic pressure and normal load mentioned in [15], the waypoint constraints, which are often defined to execute the multiple mission tasks, such as reconnaissance or multi-payloads deployment, are considered in our work. Similar to [20,14,15], sequential linearization is used in this work to address the nonlinear system dynamics. As reference [15] highlighted, the sequential linearization of system dynamics with respect to the control may cause significant undesired chatter in the control profile and even great convergence difficulty. The two control variables to be optimized in our work render more challenges than that resulting from single control variable in [15]. To cope with these challenges, equivalently alternative variables are introduced and the appropriate relaxation techniques for the control constraints are developed; moreover, the concave path constraints on heat flux, dynamic pressure, load factor and NFZs are sequentially approximated by linear inequality constraints. The major part of this paper is devoted to analytically ensuring that the relaxed problem has the same solution as the original problem. The concave constraints on waypoints are converted to several boundary constraints in the sequence of multiphase convex optimization problems, and the obstacles that result from the unknown time instants that the vehicle passes through the waypoints are avoided in our proposed multiphase convex programming method. The numerical results and comparison study are provided to validate the effectiveness and efficiency of the proposed method.

The remainder of this paper is organized as follows: Section 2 formulates the CAV's entry trajectory optimization problem and the various nonlinear constraints; Section 3 presents the main results for the reformulation, convexification and numerical solution procedure of this problem, and a rigorous theoretical analysis is presented to ensure the activity of the relaxed constraints; the numerical results and comparison study are shown in Section 4 to demonstrate the effectiveness of the proposed method; and Section 5 concludes this paper.

2. Problem formulation

This section presents the entry dynamics as well as the specific constraints used in this research, and then the continuous-time dynamic optimization problem with free final time is formulated.

2.1. CAV entry motion description

Before modeling the motion of CAV, we made the following assumptions [21]: (1) the earth is non-rotating; (2) constant gravitational acceleration; (3) small flight path angle, which implies $\sin\gamma \approx \gamma$ and $\cos\gamma \approx 1$; (4) limited control inputs (bank angle and angle of attack); (5) simple exponential atmospheric density model. Further, a Mach independent aerodynamic model taken from [22] is used in this research.

According to [19], define the normalized coefficient of lift as

$$\lambda = \frac{C_L}{C_L^*} \tag{1}$$

where C_L is the lift coefficient, and C_L^* is the lift coefficient that produces the maximum lift-to-drag ratio. In light of the aforementioned assumptions and definitions, the non-dimensional equations of CAV's motion are given as [19].

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{h} &= v\gamma \\ \dot{v} &= -Bv^2 e^{-\beta R_e h}(1+\lambda^2)/2E^* \\ \dot{\gamma} &= Bve^{-\beta R_e h}\lambda\cos\sigma - 1/v + v \\ \dot{\theta} &= Bve^{-\beta R_e h}\lambda\sin\sigma \end{aligned} \tag{2}$$

where $\dot{*} = \mathrm{d}*/\mathrm{d}t$ and t is normalized by $\sqrt{R_e/g_e}$ (R_e is the earth's radius, and g_e is the gravitational acceleration at sea level), the states x and y represent the horizontal positions normalized by the earth's radius R_e, h is the altitude normalized by R_e, v is the vehicle speed normalized by $\sqrt{R_e g_e}$, γ is the flight path angle, and θ is the heading angle. The vehicle specific constant B is defined by $B = \rho_0 R_e S_{ref} C_L^*/(2\,m)$ where ρ_0 is the atmospheric density at sea level, S_{ref} is the aerodynamic reference area, and m is the vehicle mass. In (2), the control variables are the bank angle σ and the normalized coefficient of lift λ.

Let the states in (2) be $\mathbf{x} = [x, y, h, v, \gamma, \theta]^T$, and the controls be $\mathbf{u} = [\lambda, \sigma]^T$, then rewrite (2) as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \tag{3}$$

2.2. Flight constraints

In general, path constraints, such as the heating rate, dynamic pressure, and load factor, guarantee the safe flight of a vehicle. Moreover, in many cases, waypoint and no-fly zone constraints are prescribed to execute multiple tasks and avoid international dispute. The resulting waypoints specify a series of positions that the vehicle must directly pass overhead to execute the prescribed multiple tasks, such as validation of the vehicle navigation, strikes on different locations, etc. The no-fly zone defined a region that the vehicle should not enterto avoidan international dispute. All of these flight constraints can be classified into two groups: equality and inequality constraints.

2.2.1. Equality constraints

The initial condition and the terminal constraints of CAV are determined by the prescribed entry start and the target according to the mission profile; therefore, the following equality constraints apply:

$$\mathbf{\Phi}(\mathbf{x}(t_0), \mathbf{x}_0) = \mathbf{0}, \quad \mathbf{\Psi}(\mathbf{x}(t_f), \mathbf{x}_f) = \mathbf{0} \tag{4}$$

Note that in many cases, the initial and terminal states are partially specified, for example, only the terminal horizontal position is prescribed in the research of [19]. The terminal flight path angle $\gamma(t_f)$ and heading angle $\theta(t_f)$ may be prescribed in some cases, thereby imposing additional equality constraints on the terminal state $\mathbf{x}(t_f)$. Furthermore, these additional equality constraints can be replaced by inequality constraints, while $\gamma(t_f)$ and $\theta(t_f)$ are limited within a prescribed range.

Waypoint constraints are also defined by equality constraints. Consider n_W waypoints specified by $\mathbf{wp}_i = [x_i, y_i]^T$, $(i = 1, \ldots n_W)$ in which the horizontal positions that the vehicle must pass overhead are defined. Thus, the solution of (3) on the certain time t_i, which is unknown in advance, is constrained by

$$\underset{i=1,\ldots,n_W}{\mathbf{W}(\mathbf{x}(t_i), \mathbf{wp}_i)} = \begin{bmatrix} x(t_i) - x_i \\ y(t_i) - y_i \end{bmatrix} = \mathbf{0} \tag{5}$$