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Transfer orbits to L_4 with a solar sail in the Earth-Sun system

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ABSTRACT

Solar sails are enablers for long interplanetary transfers, but also offer many advantages in Libration Point Orbits missions. The extra effect of the Solar Radiation Pressure allows a space vehicle, by changing the sail orientation, to be artificially displaced from the classical Lagrangian equilibrium points, $L_1, \ldots, L_5$, as well perturbed from the Lyapunov, Halo and Lissajous orbits that appear around them. Most of these equilibrium points are linearly unstable and have stable and unstable invariant manifolds associated with them. In this paper we explore the possibilities that these invariant manifolds offer to navigate in a natural way around a circular, restricted, three-body system. We take the Earth-Sun Restricted Three Body Problem as a model and, for different fixed sail orientations, we compute the stable and unstable manifolds associated with the equilibrium points of the system. We find natural trajectories that allow the vehicle to move around the family of equilibria in a controlled way and to go from a region close to L_1 or L_2 to a region close to L_4.

1. Introduction

A solar sail is a spacecraft propulsion device that employs large reflecting surfaces to exploit solar radiation pressure (SRP), enabling a constant acceleration and potentially being able to thrust a spacecraft indefinitely. This feature offers the opportunity to combine low-cost operations with long-term missions.

The successful deployment of a solar sail on IKAROS (December 2010) by JAXA,[1] NanoSail-D2 (January 2011) by NASA,[2] and recently LightSail (June 2015) by the Planetary Society[3] have validated the concept of solar sailing. All three missions where focused on demonstrating the technology of solar sailing, up to the date there has not been any scientific mission using solar sails. Nevertheless, many studies on this novel propulsion system are being conducted. In astrodynamics research, solar sails are being considered for different mission scenarios. One example is the Sunjammer mission, an enhanced warning mission to detect solar magnetic storms, where the solar sail allows the vehicle to be displaced from the classical Lagrangian equilibrium points [1]. Alternatively, drag sails are considered to accelerate the de-orbiting of LEO satellites [2], and sails have been proposed as an end-of-life option in Libration Point Orbits (LPO) [3]. Solar sail spacecraft have also been suggested for a low-cost multi-NEO rendezvous mission; the mission could visit many asteroids and be flexible in the selected destinations [4].

In this paper we explore the structure of invariant manifolds that exist in the Restricted Three Body Problem (RTBP) and use it to derive transfer orbits between different regions in the phase space. The stable and unstable invariant manifolds in the RTBP have already been used to provide inexpensive transfer orbits between the Earth and a Halo orbit in the Genesis mission, and their potential using chemical propulsion systems has been widely studied [5–12].

To model the dynamics of a solar sail in the Earth-Sun system we consider the RTBP and incorporate the effect of the solar radiation pressure due to the solar sail (RTBPS). In an idealised model, the acceleration given by the solar sail depends on three parameters: the sail lightness number (β) that measures the sail efficiency and two angles (α, δ) that measure the orientation of the sail. It is well known that the extra effect of the solar sail in the RTBPS allows one to "artificially" displace the Lagrangian equilibrium points ($L_{1,\ldots,5}$) resulting in a two-dimensional (2D) family of equilibrium points for a given β [13,14,15,16].

These new equilibrium points offer privileged positions in the phase space for observational missions and have been proposed for missions such as Sunjammer [13,1] and Polar Sitter [17,18]. The Sunjammer mission (also known as Geostorm) would place a sailcraft at an equilibrium point closer to the Sun than the classical L_1 and displaced the vehicle about 5° from the Sun-Earth line, enabling observations of the Sun's geomagnetic field while having a constant communication with the Earth. This positioning would enable the sail observatory to alert ground stations of Geo-magnetic storms, potentially doubling the

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[1] http://www.isas.jaxa.jp/e/enterp/missions/ikaros/index.shtml.
[2] http://www.nasa.gov/mission_pages/smallsats/nanosaild.html.
[3] http://sail.planetary.org.

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actual alert time from ACE, which is orbiting on a Halo orbit around L_1. Alternatively, Halo orbits around the displaced L_1 could also be considered for such a mission [1]. On the other hand, the Polar Sitter (or Polar Observer) mission proposes to use the solar sail to find an equilibrium point above or below the ecliptic plane, high enough to enable continuous monitoring of the Earth's Polar regions. This could solve the poor temporal resolution of polar observations from highly inclined LEO. Alternatively, constellations at vertical Lyapunov orbits and other artificial displaced orbits by solar sails and hybrid propulsion systems have also been proposed [18].

We have computed these families of equilibrium points for different sail configurations ($\beta = 0.01, 0.02, 0.03, 0.04$ and 0.05) and classified them with regard to their stability and other properties. Many of these artificial points are unstable and have stable and unstable manifolds associated with them. These invariant manifolds are used as natural transfer trajectories from one place to another in the phase space. Most of these equilibrium points possess families of periodic Lyapunov and Halo orbits in their respective vicinities [19]. These orbits are also unstable and their invariant manifolds can also be used to generate transfer trajectories. However, in this paper we focus the study on the options that the stable and unstable manifolds related to the equilibrium points, and their families, provide. As we will see this design space is very rich. This paper is only a preliminary study to show that these transfer trajectories can be obtained using a solar sail.

We have organised this paper as follows: in Sections 2 and 3 we define the dynamical model. In Section 4 we discuss the Jacobi constant and its implications in our model. In Section 5 we describe the families of equilibrium points when we change the sail orientation, as well as their properties. In Section 6 we describe the invariant manifolds connected to the artificial equilibrium points and focus on three different mission applications. The first application (Section 6.1) employs a linear approximation of these manifolds to generate a surfing strategy to drift along the family of equilibria in a controlled way. The second application (Section 6.2) is to study the feasibility of transferring a solar sail from the L_1 region to the L_2 region. The final application (Section 6.3) is to use the solar sail to gain enough energy to transfer from L_1 or L_2 to a neighbourhood of L_4. Finally, in Section 7 we show a brief study on the cost of a solar sail transfer to L_4 from the L_1 and L_2 vicinity.

2. Equations of motion

To describe the motion of a solar sail spacecraft in the Earth-Sun system we consider as a model the Circular Restricted Three Body Problem and incorporate Solar Radiation Pressure (SRP) acting on the solar sail spacecraft. We assume that the Earth and the Sun are point masses moving around their common barycentre circularly because of their mutual gravitational attraction. The solar sail spacecraft is assumed to be a massless particle that does not affect the motion of the two primaries but is affected by their gravitational attraction as well as by the SRP.

We normalise the units of mass, distance and time, so that the total mass of the system is 1, the Earth-Sun distance is 1 and the period of one Earth-Sun revolution is 2π. In these units the universal gravitational constant G=1, the mass of the Earth is $\mu = 3.0034806 \times 10^{-6}$, and $1 - \mu$ corresponds to the mass of the Sun. We use a rotating reference system with the origin at the centre of mass of the Earth-Sun system such that the Earth and the Sun are fixed on the x-axis (with its positive direction towards the Sun), the z-axis is perpendicular to the ecliptic plane and the y-axis completes an orthogonal positive oriented reference system [20].

With these assumptions, the equations of motion in the rotating reference system are:

$$\begin{aligned}
\ddot{x} - 2\dot{y} &= x - (1-\mu)\frac{x-\mu}{r_{ps}^3} - \mu\frac{x-\mu+1}{r_{pe}^3} + a_x,\\
\ddot{y} + 2\dot{x} &= y - \left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right)y + a_y,\\
\ddot{z} &= -\left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right)z + a_z,
\end{aligned} \tag{1}$$

where $\mathbf{a} = (a_x, a_y, a_z)$ is the acceleration given by the solar sail, and $r_{ps} = \sqrt{(x-\mu)^2 + y^2 + z^2}$, $r_{pe} = \sqrt{(x-\mu+1)^2 + y^2 + z^2}$ are the Sun-sail and Earth-sail distances respectively.

3. The solar sail acceleration

The acceleration provided by the solar sail depends on its efficiency, defined by the sail lightness number β, and its orientation, parameterised by two angles α, δ. In this paper we consider the simplest model for a solar sail; that is, we assume it to be flat and perfectly reflecting. Hence, the acceleration due to the SRP is in the normal direction to the surface of the solar sail. For more realistic model one should include the effect due to the absorption of the photons by the surface of the sail, as well as the specular and diffusive reflection [21,22], more realistic models also include surface imperfections [23,24]. In this case, we must add an extra component in the transverse direction to the sail that will change the efficiency of the sail and the direction of the acceleration vector. These changes can be quantified and are small for a high-performance sail whose orientation is close to perpendicular with respect to the Sun-sail line.

The force produced by the reflected photons is given by $\mathbf{F_r} = 2PA\langle \mathbf{n}, \mathbf{r}_s\rangle^2\mathbf{n}$, where $P = P_0(R_0/R)^2$ is the SRP magnitude at a distance R from the Sun (being $P_0 = 4.563$ N/m^2 the SRP magnitude at $R_0 = 1$ AU), A is the frontal area of the solar sail, $\mathbf{r}_s$ is the Sun-sail direction and $\mathbf{n}$ is the normal direction to the surface of the sail (both unit vectors). Notice that unless the sail is perpendicular to the Sun-sail line the vectors $\mathbf{n}$ and $\mathbf{r}_s$ are different. As the SRP is proportional to the inverse square of the distance to the Sun, it is common to write its effect as a correction of the Sun's gravitational attraction [13]:

$$\mathbf{a} = \beta\frac{(1-\mu)}{r_{ps}^2}\langle \mathbf{r}_s, \mathbf{n}\rangle^2\mathbf{n}, \tag{2}$$

where β corresponds to the sail lightness number, which accounts for the sail's effectiveness. One can also interpret β as the ratio between the solar sail acceleration and the gravitational attraction. According to McInnes [13].

$$\beta = \sigma^*/\sigma, \quad \sigma^* = \frac{2P_0R_0^2}{Gm_s} = 1.53 \text{ g/m}^2,$$

where $\sigma = m/A$ is the inverse of the area-to-mass ratio of the solar sail. We call characteristic acceleration, a_0, to the acceleration experienced by the sailcraft at 1 AU and face-on to the Sun (i.e., $a_0 = \beta\ Gm_{sun}/1\text{AU}^2$). This means that a sail lightness number $\beta = 0.03$ corresponds to a characteristic acceleration $a_0 = 0.179804$ mm/s^2. Moreover, if we have a total spacecraft mass of 10 kg the solar sail area must be approximately 14×14 m^2 for a sail lightness number $\beta = 0.03$. Table 1 lists, for different sail lightness numbers β, the corresponding inverse of the area-to-mass ratio (σ), the characteristic acceleration (a_0) and the size of the solar sail for 10 kg of total spacecraft mass.

The sail orientation is given by the normal direction to the surface of the sail, $\mathbf{n} = (n_x, n_y, n_z)$, and is parameterised by two angles α and δ that measure the orientation of $\mathbf{n}$ with respect to the Sun-sail direction $\mathbf{r}_s = (x-\mu, y, z)/r_{ps}$. Following [14], we take $\{\mathbf{r}_s, \mathbf{p}, \mathbf{q}\}$ an orthonormal reference frame centred at the solar sail, where $\mathbf{p} = \frac{\mathbf{r}_s \times \mathbf{z}}{|\mathbf{r}_s \times \mathbf{z}|}$ and $\mathbf{q} = \frac{(\mathbf{r}_s \times \mathbf{z}) \times \mathbf{r}_s}{|(\mathbf{r}_s \times \mathbf{z}) \times \mathbf{r}_s|}$. Then we define $\mathbf{n} = \cos\alpha\ \mathbf{r}_s + \sin\alpha\cos\delta\ \mathbf{p} + \sin\alpha\sin\delta\ \mathbf{q}$. Hence,