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An accurate procedure for estimating the phase speed of ocean waves from observations by satellite borne altimeters

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ABSTRACT

Observations of sea surface height (SSH) fields using satellite borne altimeters were conducted starting in the 1990s in various parts of the world ocean. Currently, a long period of 20 years of calibrated and accurate altimeter observations of Sea Surface Height Anomalies (SSHA) is publically available and ready to be examined for determining the rate of westward propagation of these anomalies, which are interpreted as a surface manifestation of linear Rossby waves that propagate westward in the ocean thermocline or as nonlinear eddies. The basis for estimating the speed of westward propagation of SSHA is time-longitude (Hovmöller) diagrams of the SSHA field at fixed latitude. In such a diagram the westward propagation is evident from a left-upward tilt of constant SSHA values (i.e. contours) and the angle between this tilt and the ordinate is directly proportional to the speed of westward propagation. In this work we use synthetically generated noisy data to examine the accuracy of three different methods that have been separately used in previous studies for estimating this slope (angle) of the time-longitude diagram: The first is the application of Radon transform, used in image processing for detecting structures on an image. The second method is the application of 2D Fast Fourier Transform that yields a frequency-wavenumber diagram of the amplitudes so the frequency and wavenumber where the maximum amplitude occurs determine the phase speed i.e. the slope. The third method constitutes an adaptation of Radon transform to a propagating wave in which structures of minimal variance in the image are identified. The three methods do not always yield the same phase speed value and our analysis of the synthetic data shows that an estimate of the phase speed at any given latitude should be considered valid only when at least two of the methods yield the same value. The relevance of the suggested procedure to observed signals is verified by applying it to observed SSHA signals in the ocean.

1. Introduction

Rossby waves are westward-propagating, low-frequency, planetary waves that provide the main low frequency mechanism for the mutual adjustment of pressure, momentum and mass in the ocean and atmosphere. These waves owe their existence to the latitudinal variation of the Coriolis frequency which originates from the rotation of Earth about its polar axis and is proportional to the sine of the latitude [1]. Observations of these large scale waves in the ocean could not have been carried out prior to the launch of Earth observing satellites. The analysis of observations of Sea Surface Height Anomalies (SSHA), i.e., the deviation of the Sea Surface Height from its mean value at any given point in the ocean, was carried out since the 1990s in various parts of the world ocean by various satellite borne altimeters. These observations showed a ubiquitous and pronounced westward migration of low amplitude (a few centimeters) SSHA which led to their interpretation as a surface manifestation of Rossby waves that propagate westward in the ocean thermocline (a range of depths below the ocean surface, a few hundred meters thick, where the temperature drops sharply with depth). Previous studies of the westward propagation of observed SSHA in mid-latitudes have all yielded rates of westward propagation that are faster than the phase speeds predicted by the traditional Rossby wave theory. Explanations for these underestimates by the traditional theory were proposed based on, e.g., the addition of mean zonal flows in the equations [2], the effect of bottom topography [3] or a combination of these effects [4]. These past studies (as well as other suggested mechanisms) have modified the phase speed of the traditional wave theory but these modifications bridged only part of the gap and in selected locations. Recent studies (e.g., [5,6]) argue that nearly all the variability of SSHA originates from nonlinearity of mesoscale eddies rather than surface manifestations of linear Rossby waves in the thermocline. However, this change of view has no effect on

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the estimate of the westward propagation speed since these eddies propagate westward at the phase speed of Rossby waves [6,7]. Therefore, it is still of importance to obtain reliable estimates of the propagation speed of the observed phenomena for making inferences about the dynamics involved in these oceanic features.

Given the importance of obtaining reliable estimates for the phase speed of Rossby waves (or nonlinear eddies) from satellite SSHA observations, we compare in this paper three different methods that have been separately used in previous studies for estimating the phase speed. First, we examine the three methods on a synthetic noisy signal and from this examination deduce an accurate procedure of interpreting satellite data by combining at least two of these independent methods. This procedure is then applied to observed SSHA signals in the ocean to verify its higher accuracy when the data is contaminated by observed noise rather than white noise. The method proposed by Polito and Liu [8] in which filtering in time and space is applied to the raw data prior to the calculation of the Radon transform is not studied here since there is no uniform or acceptable way of filtering of the raw data.

2. Methods

2.1. Methods for estimating observed phase speed

The basis for estimating the speed of westward propagation of SSHA is time-longitude (Hovmöller) diagrams of the SSHA field at fixed latitude. In this diagram the westward propagation is evident from the left-upward tilt of constant SSHA values i.e., same color (level) contours. The angle between this tilt and the ordinate is directly proportional to the speed of westward propagation. The diagram provides a time-series of the SSHA changes at fixed longitude and a longitude variation series at any particular time so Fast Fourier Transforms can be easily calculated in time and longitude to yield the frequency and zonal wavenumber spectra of observed SSHA.

2.2. Radon transform

A common objective algorithm frequently used (e.g., ref. [9–11]) to calculate the phase speed of waves out of time-longitude diagrams employs the Radon transform which is used in image processing for detecting structures on any digital image (see details in e.g., ref [12]). Briefly, the Radon transform is a function defined on straight lines L in the (x, y) plane that are inclined at an angle θ relative to the ordinate (i.e., $\theta \pm 90°$ relative to the abscissa) and displaced a distance s from an arbitrarily chosen origin (see Fig. 1). Note that (s, θ) are not polar coordinates since all the points (x, y) along the line L satisfy $s = x\cos\theta + y\sin\theta$ where s is the distance of the line L from the origin that should not be confused with the distance $r = \sqrt{x^2 + y^2}$ of a single point from the origin. The Radon transform of a two dimensional function $f(x, y)$ that describes the intensity of an image at any (x, y) point, such as SSHA values in a given (λ, t) domain, is the integral of $f(x, y)$ along L.

The Radon transform maps the (x, y) domain to the (s, θ) space and each (s, θ) point corresponds to a line L in the spatial domain (x, y). This transform is applied to the time-longitude diagram by integrating (i.e., summing up) the SSHA values along each line L having an angle θ and a distance s. For each angle θ we sum the squares of the values of the integrals calculated in the previous step along all lines having the same θ (but having different distance s). This second step yields values that typify θ only and the angle at which this sum-of-squares attains its maximum is the most accurate estimate for the orientation of structures with the same SSHA value on the time-longitude diagram. The sought westward propagation speed is proportional to the tangent of this preferred θ. Note that in order to minimize the effect of few very high entries on the sum of squares, we apply the Radon transform to a modified time-longitude diagram where the signal is scaled on the [0,1] interval and the mean of the scaled signal is subtracted.

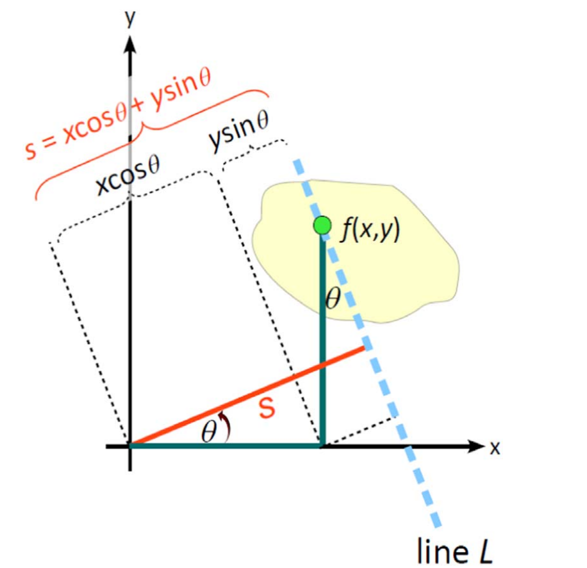

Fig. 1. The Radon transform of $f(x, y)$ is the integral of f along the line L oriented at angle θ relative to the ordinate and located at distance s from the origin.

2.3. Variance

We also examine a new algorithm (suggested by [13]) that constitutes an adaptation of Radon transform to the particular case where the sought structures are phase contours. Instead of looking at integrals of amplitude values along lines, in this new algorithm the **variance** of amplitude values along lines is calculated. Each point in the (s, θ) space corresponds in this algorithm to the variance of SSHA values along L (rather than sum in the Radon transform) and for each angle θ we average the variances at all values of s. The westward propagation speed is then determined by the angle θ at which the mean of variances is minimal.

2.4. Two dimensional Fast Fourier Transform (2D FFT)

An independent method commonly used [e.g., 14–16] to obtain the observed phase speed is the application of the 2D Fast Fourier Transform (2D FFT) to the time-longitude diagram to get a frequency-wavenumber (i.e., ω, k) diagram of the signal's amplitude. The phase speed is obtained by locating the values of ω and k where the amplitude is maximal (i.e., maximum spectral coefficient) and calculating the phase speed C $(=\omega/k)$ at this point of maximum spectral coefficient. Alternatively, the directionality of the spectral coefficients in the (ω, k) diagram can be found by sweeping over all lines that pass through the origin and are inclined at angles ranging from 0° to 180° relative to the abscissa (i.e., lines of uniform ω/k values). The value of C is then determined as the slope of the line of maximal sum-of-squares of spectral coefficients ("energy"). Note that in this method the resolution of ω and k increases with the size of the time-longitude domain.

2.5. Comparison between the three methods

In order to compare these three methods we generated a synthetic signal made up of known phase speeds and noise. The phase of a single sine wave of the form $\sin(x\text{-}ct)$ where c is the wave propagating speed, is constant along lines in (x, t) space that satisfy $x\text{-}ct$=constant. Thus, the Hovmöller diagram is made up of straight lines with slopes of $1/c$ i.e., lines inclined at slope-angles given by $\arctan(1/c)=\text{arccot}(c)$