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Invariant manifold connections via polyhedral representation

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ABSTRACT

In recent years, manifold dynamics has assumed an increasing relevance for analysis and design of low-energy missions, both in the Earth-Moon system and in alternative multibody environments, and several space missions have already taken advantage of the results of the related studies. Recent efforts have been devoted to developing a suitable representation for the manifolds, which would be extremely useful for mission analysis and optimization. This work describes and uses a recently-introduced, intuitive polyhedral interpolative approach for each state component associated with manifold trajectories, both in two and in three dimensions. A grid of data, coming from the numerical propagation of a finite number of manifold trajectories, is used. This representation is employed for some invariant manifolds, associated with the two planar Lyapunov orbits at the collinear libration points located in the proximity of the Moon, and with a three-dimensional Halo orbit. Accuracy is evaluated, and is proven to be satisfactory, with the exclusion of limited regions of the manifolds. This paper describes the use of this polyhedral representation for the detection of homoclinic and heteroclinic connections. In particular, a variety of homoclinic trajectories connected with two Lyapunov orbits are detected. Then, the polyhedral interpolating technique is successfully employed for the determination of heteroclinic connections between the manifolds associated with the Lyapunov orbits. Lastly, near-homoclinic connections between the manifolds emanating from a Halo orbit are detected. The results achieved in this paper prove utility and effectiveness of the polyhedral interpolative technique for detecting manifold connections in the circular restricted three-body problem, and represent the premise for its application to space mission analysis involving invariant manifold dynamics.

1. Introduction

Analysis and design of low-energy trajectories to the Moon has attracted a strong interest in the last decades. Exterior and interior transfers, based on the transit through the regions where the collinear libration points L_1 and L_2 are located, have been studied for a long time and some missions have already taken advantage of the results of these studies. Examples are the European Smart-1 mission [1], the Japanese Hiten mission [2], and the NASA missions Genesis [3], Artemis [4], and GRAIL [5]. These missions exploit special classes of unstable periodic orbits that are proven to exist in the context of the circular restricted three-body problem.

In general, unstable orbits are associated with stable and unstable invariant manifolds, which are composed of all the trajectories asymptotically departing from or converging into the periodic orbit of interest. They are ideally traveled without any propellant consumption, albeit a modest fuel amount is concretely needed for manifold injection and orbit maintenance. The use of invariant manifolds allows designing a great variety of space missions, associated with modest propellant budgets. Similar concepts have been developed also for possible missions in the Jovian [6] and Saturnian systems [7]. Transfer trajectories employing invariant manifolds have been investigated by Marsden et al. [8], Gomez and Masdemont [9], and Davis et al. [10]. Marson et al. [11] addressed the problem of deploying a small satellite constellation around the Moon by employing invariant manifolds. Giancotti et al. [12,13] and Pontani and Teofilatto [14,15] proposed a useful cylindrical isomorphic mapping for investigating manifold dynamics in the planar circular restricted three-body problem.

Recent efforts have been devoted to developing a suitable representation for the trajectories that form the manifolds, which would be extremely useful for mission analysis and optimization. Typically, the invariant manifold trajectories are numerically propagated along the directions associated with the eigenvectors of the monodromy matrix. This is a computationally expensive task, and an interpolative approach is desirable for mission design and optimization. With this intent, Howell et al. [16] proposed two approaches: the first employs splines, while the second technique is based on using a cell structure for the purpose of approximating the manifold. Moreover, Masdemont [17]

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employed higher order expansions around libration point orbits, whereas Ozimek and Howell [18] and Martin and Conway [19] used bicubic spline interpolation.

The manifolds are topologically two-dimensional, because each point belonging to them (and the position and velocity) can be identified by means of two quantities: (i) the injection point along the periodic orbit, and (ii) the time of flight on the manifold. This fundamental property discloses the possibility of representing the position and velocity components as functions of two variables. This means that geometrically each component is associated with a surface, which nevertheless cannot be described in any analytical, closed form.

The polyhedral interpolation technique [20–22] represents an intuitive approach that allows expressing each state component as a piecewise linear function of the two mentioned quantities. Its effectiveness in determining optimal impulsive and low-thrust transfers was proven in a recent paper [22].

The work that follows is intended to extend the use of the polyhedral representation to the detection of homoclinic and heteroclinic connections. These are special trajectories that belong simultaneously to two distinct manifolds. If these two manifolds emanate from the same periodic orbit, then the connection is termed homoclinic. Conversely, if the two manifolds are associated with two distinct periodic orbits, the connection is referred to as heteroclinic. Specifically, after evaluating the accuracy of the representation at hand, three applications are illustrated, i.e.,

(a) identification of the symmetric homoclinic connections associated with the left and right manifolds of the Lyapunov orbit at L_1 and with the left manifolds of the Lyapunov orbit at L_2;
(b) detection of the heteroclinic connections between the manifolds associated with the Lyapunov orbits at L_1 and at L_2;
(c) determination of near-homiclinic paths emanating from a Halo orbit.

In the end, this work has the objective of describing and applying this new representation for the invariant manifolds for the detection of manifold connections, pointing out its simplicity, ease of use, and satisfactory accuracy.

2. Circular restricted three-Body problem

The circular restricted three-body problem (CR3BP) models the dynamics of three bodies with masses m_1, m_2, and m_3, under the assumption that $m_1 > m_2 >> m_3 \approx 0$. This means that the mass of the third body (i.e. the spacecraft), m_3, is considered negligible, whereas the remaining massive bodies, termed the primaries, describe counterclockwise circular orbits around their center of mass. This model approximates the dynamics of a spacecraft subject to the gravitational attraction of the Earth and of the Moon (that represent the primaries), under the assumption that the Moon motion around the Earth is planar.

The problem is conveniently described in synodic coordinates, which represent a coordinate system rotating with the two primaries. The x-axis connects the two primaries and is directed from the Earth to the Moon, the y-axis lies in the plane of their motion, and the z-axis points toward the direction of angular momentum. This problem is analyzed by employing canonical units, which represent a set of convenient normalized units. The distance unit DU is the (constant) distance between the two primaries (1 DU =384400 km), whereas the time unit TU is such that the two primaries complete a single orbit in a period equal to 2πTU (1 TU =375190 s). If μ_E and μ_M denote respectively the gravitational parameter of the Earth and of the Moon, these definitions of TU and DU imply that $\mu_E + \mu_M = 1\text{DU}^3/\text{TU}^2$. After introducing the parameter $\mu: =\mu_M/(\mu_E + \mu_M)$ (=0.012155), it is straightforward to rewrite the gravitational parameters of the two primaries as $\mu_E = 1 - \mu$ and $\mu_M = \mu\,(\text{DU}^3/\text{TU}^2)$. Their positions along the x-axis are $x_E = -\mu$ and $x_M = 1 - \mu$ (DU).

2.1. Equations of motion and Jacobi integral

In the synodic reference frame, let (x, y, z) and (v_x, v_y, v_z) denote the coordinates of position and velocity. The equations of motion [23] of the CR3BP are

$$\dot{x} = v_x \tag{1}$$

$$\dot{y} = v_y \tag{2}$$

$$\dot{z} = v_y \tag{3}$$

$$\dot{v}_x = \frac{\partial \Omega}{\partial x} + 2v_y \tag{4}$$

$$\dot{v}_y = \frac{\partial \Omega}{\partial x} - 2v_x \tag{5}$$

$$\dot{v}_z = \frac{\partial \Omega}{\partial z} \tag{6}$$

where

$$\Omega: =\frac{x^2 + y^2}{2} + \frac{1-\mu}{r_E} + \frac{\mu}{r_M} \tag{7}$$

$$r_E\ : = \sqrt{(x+\mu)^2 + y^2 + z^2} \qquad r_M\ : = \sqrt{(x+\mu-1)^2 + y^2 + z^2} \tag{8}$$

Eqs. (1)–(6) can be written in compact form as

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}), \quad \mathbf{X}: =[x \ \ y \ \ z \ \ v_x \ \ v_y \ \ v_z]^T \tag{9}$$

It is relatively straightforward to demonstrate that an integral exists for this dynamical system: the Jacobi integral, whose value is referred to as the Jacobi constant and denoted with C [23],

$$C = 2\Omega - (v_x^2 + v_y^2 + v_z^2) \tag{10}$$

The value of C remains unchanged, and is associated with the energy of the dynamical system.

The Jacobi integral represents an invariant quantity and can be regarded as an equality constraint for the dynamical system under consideration. This circumstance implies that once C is specified, only five out of six initial conditions (for the components of position and velocity) are independent. An elegant way of incorporating the Jacobi constant in the dynamic equations consists in extending to three dimensions the two-dimensional Birkhoff's equations [15,24]. After introducing the angles γ and β illustrated in Fig. 1, due to Eq. (10) the three components v_x, v_y, and v_z can be rewritten as

$$v_x = \sqrt{2\Omega - C}\ \cos\beta\,\cos\gamma \tag{11}$$

$$v_y = \sqrt{2\Omega - C}\ \cos\beta\,\sin\gamma \tag{12}$$

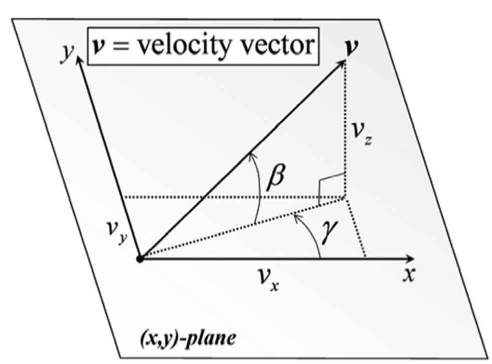

Fig. 1. : Synodic reference frame and velocity angles γ and β.