Acta Astronautica 133 (2017) 1–13

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/actaastro

Thermoelastic buckling analysis of pre-twisted functionally graded beams with temperature-dependent material properties

Amin Ghorbani Shenas[a], Parviz Malekzadeh[b,*], Sima Ziaee[a]

[a] Department of Mechanical Engineering, Yasouj University, Yasouj 75914-353, Iran
[b] Department of Mechanical Engineering, Persian Gulf University, Bushehr 7516913798, Iran

ARTICLE INFO

Keywords:
Pre-twisted beams
Functionally graded materials
Third-order shear deformation theory
Thermal buckling
Chebyshev-Ritz method

ABSTRACT

As a first endeavor, the thermal buckling behavior of pre-twisted functionally graded (FG) beams with temperature-dependent material properties is investigated. The governing stability equations are derived based on the third-order shear deformation theory (TSDT) in conjunction with the adjacent equilibrium state criterion under the von Kármán's nonlinear kinematic assumptions using the Chebyshev-Ritz method. The Chebyshev polynomials multiplied with some suitable boundary functions are used as the basis functions, which allow one to analyze the beams with different boundary conditions. The extracted system of nonlinear algebraic eigenvalue equations is solved iteratively to obtain the critical temperature rise. The convergence behavior together with accuracy of the solution method and the correctness of formulation are demonstrated through different examples. Then, the influences of the linear and nonlinear variation of the angle of twist along the beam axis, the value of twist angle, length-to-thickness ratio, thickness-to-width ratio, material gradient index and temperature dependence of material properties on the critical temperature rise of the pre-twisted FG beams under different boundary conditions are investigated. It is shown that the pre-twist angle increases the thermal buckling resistance of the pre-twisted FG beams, but the temperature dependence of material properties reduces it.

1. Introduction

The continuous and smooth spatial variations in the composition and material properties provide enhanced thermo-mechanical characteristics for the functionally graded materials (FGMs) with respect to the conventional composite materials such as the thermal protection from ablation and elimination of stress concentration [1]. The structural elements made of these advanced materials such as beams, plates and shells have a wide range of engineering applications; for example, in space vehicles, aircrafts, automobiles, nuclear power plants, combustion chambers, turbine blades, etc. Usually these structural elements work in thermal environment and consequently are subjected to thermal loadings. The induced thermal loading together with the mechanical restraints at their boundaries can cause the thermal buckling of these structural elements as one of their important failure modes [2–13]. Hence, the thermal stability study is essential in providing a functional guideline for their accurate design.

The thermal buckling analysis of FG beams has continuously attracted the attention of researchers in the last years; see for examples Refs. [2–14]. However, in spite of the evident importance for technical applications of the pre-twisted FG beams, to the best of authors' knowledge, neither thermal nor mechanical buckling behavior of these structural elements has been studied yet. In the following some of the more relevant works to the topic of this paper are briefly reviewed.

Li et al. [2] analyzed the thermal buckling and post-buckling of FG Timoshenko beams subjected to transversely non-uniform temperature. By using a shooting method, they numerically solved the nonlinear boundary value problem of beams with fixed-fixed edges. Anandrao et al. [3] investigated the thermal post-buckling behavior of uniform slender FG beams using the classical Rayleigh-Ritz method and the finite element method independently. The von-Karman strain-displacement relations were employed to account for moderately large deflections of FG beams. Simply supported and clamped beams with axially immovable ends were considered. Kiani and Eslami [4] studied the thermal buckling of FG beams under various types of thermal loading based on the Euler–Bernoulli beam theory. They obtained the closed form solution for beams with combination of clamped, roller and simply supported edges. Wattanasakulpong et al. [5] employed an improved third-order shear deformation theory to investigate the thermal buckling and free vibration of the FG beams. They applied

* Corresponding author.
E-mail addresses: p_malekz@yahoo.com, malekzadeh@pgu.ac.ir (P. Malekzadeh).

http://dx.doi.org/10.1016/j.actaastro.2017.01.007
Received 30 November 2016; Received in revised form 25 December 2016; Accepted 6 January 2017
Available online 07 January 2017
0094-5765/

the Ritz method to solve the eigenvalue problems associated with the thermal buckling and vibration of FG beams for various types of immovable boundary conditions. Fallah and Aghdam [6] presented the thermo-mechanical buckling and large amplitude free vibration analysis of FG beams on nonlinear elastic foundation. They utilized von Kármán's assumptions together with Euler–Bernoulli theory to derive the nonlinear governing equation. Galerkin's method was employed to obtain closed form solution for the critical temperature of the FG beams. Kiani and Eslami [7] presented the thermo-mechanical buckling of temperature-dependent FG beams based on the Timoshenko beam theory. They analytically derived a nonlinear algebraic equation for the critical temperature rise of the FG beams under different boundary conditions and solved it using an iterative procedure. Esfahani et al. [8] examined the thermal buckling and post-buckling analysis of FG beams with temperature-dependent material properties resting on a non-linear elastic foundation. The governing equations were derived based on the FSDT of beams under the von Kármán's kinematic assumptions. They employed the generalized differential quadrature method (GDQM) to discretize the equilibrium equations subjected to different types of boundary conditions such as clamped, simply supported, and rolled edges in space domain. Ghiasian et al. [9] studied the static and dynamic buckling of FG beams with initial imperfection under uniform temperature rise loading and uniform compression based on the classical beam theory. The three-parameter elastic foundation with hardening/softening cubic nonlinearity which acts in tension as well as in compression was considered. The nonlinear governing equations of static and dynamic buckling were solved via the multi-term Galerkin method and Hoff-Simitses criterion, respectively. The dynamic buckling and imperfection sensitivity of the FG Timoshenko beams subjected to sudden uniform temperature rise were investigated by Ghiasian et al. [10]. They utilized the conventional multi-term Ritz method and the Newmark family of time approximation scheme to convert the governing equations into a set of nonlinear algebraic equations, which was solved by means of Newton–Raphson iterative scheme. Paul and Das [11] studied the post-buckling behavior of FG Timoshenko beams under non-uniform temperature rise across the beam thickness using the Ritz method. The effect of geometric nonlinearity at large post-buckled configuration was incorporated using von Kármán's non-linear kinematic assumptions. They solved the system of non-linear algebraic equations by applying Broyden's algorithm. Sun et al. [12] investigated the buckling and post-buckling deformations of FG Timoshenko beams resting on a two-parameter nonlinear elastic foundation and subjected to temperature rise using the shooting method. Trinh et al. [13] employed the state space approach to obtain an analytical solution for the free vibration and buckling of FG beams with various boundary conditions under mechanical and thermal loads. They formulated the problems based on a third-order shear deformation theory.

In the above reviewed important research works, the FG beams without pre-twist angle have been analyzed. As a first attempt, in this work, the thermal buckling behavior of the pre-twisted FG beams composed of metal and ceramic with temperature-dependent material properties is studied. The global behavior of the beams is accurately modeled by using the Reddy's third-order shear deformation theory of beams under the von Kármán's nonlinear kinematic assumptions. In addition to better model the variation of transverse shear deformations, this theory needs not shear correction factor which is a challenging parameter for the FG structural elements under flexural deformation. The system of nonlinear algebraic eigenvalue equations corresponding to the critical temperature rise is derived by using the adjacent equilibrium criterion together with the Chebyshev-Ritz method. It should be mentioned that the computational efficiency and accuracy of the Chebyshev–Ritz method for solving complicated structural problems have been established in the previous research works [15–19]. After validating the approach, the influences of the different geometrical and material parameters on the critical temperature rise of the pre-twisted FG beams under different boundary conditions are investigated.

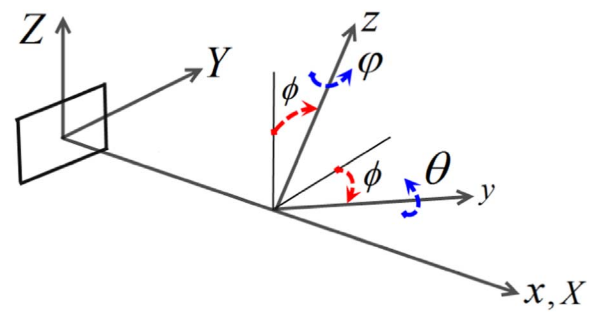

Fig. 1. The global and local coordinate system together with the rotation components of the pre-twisted beams.

2. Mathematical modeling

The geometry, global coordinate system *X-Y-Z* and local coordinate system *x-y-z* (in the lower cases) of the pre-twisted FG beams under consideration are shown in Fig. 1. The global X and local x axes are coincident, both passing through the centroid and are perpendicular to the beam cross-section, and hence, represent the axis of twist of the beam cross section (see Fig. 2). As it is obvious, the FG beam has a length L, constant thickness h, width b, and the angle of twist along the x-axis φ (see Fig. 1). Furthermore, it is assumed that the volume fractions of the material phases (i.e., metal and ceramic phases) vary continuously in the thickness direction of the FG beam. The governing equations and the solution method are presented in the following subsections.

2.1. The thermoelastic stability equations

In this study, the thermal load after which the out-of-plane displacement components of the pre-twisted FG beams become much sensitive to the in-plane thermal load is defined as the critical thermal load and the corresponding temperature rise is called the critical temperature rise [13]. To obtain this critical thermal load (or the corresponding critical temperature rise), the strain and stress components prior to this equilibrium state should be evaluated exactly or in an approximate manner. The displacement components of the FG beam at this primary equilibrium state are denoted by u_{01}, u_{02} and u_{03} in the x-, y- and z-directions, respectively. According to the adjacent equilibrium criterion [20], the displacement components at the perturbed neighboring equilibrium state become $u_{01} + u_1$, $u_{02} + u_2$ and $u_{03} + u_3$, respectively. On the other hand, it is shown that for the isotropic thin-to-moderately thick FG beams under in-plane thermal load with axially immovable edges (i.e., $u_{01} = 0$ at x=0 and L), the stress components due to the in-plane thermal loading at the primary equilibrium state (with displacement components u_{0i}) can be approximated as [4,7],

$$\sigma_{xx}^T \approx E\alpha\Delta T,\ \sigma_{xy}^T \approx 0,\ \sigma_{xz}^T \approx 0 \qquad \text{(1a-c)}$$

where E and α are the Young's modulus and the thermal expansion

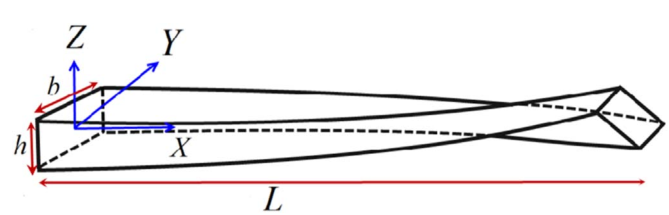

Fig. 2. The geometry and global coordinate system of the pre-twisted FG beams.