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Instantaneous GNSS attitude determination: A Monte Carlo sampling approach

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ABSTRACT

A novel instantaneous GNSS ambiguity resolution approach which makes use of only single-frequency carrier phase measurements for ultra-short baseline attitude determination is proposed. The Monte Carlo sampling method is employed to obtain the probability density function of ambiguities from a quaternion-based GNSS-attitude model and the LAMBDA method strengthened with a screening mechanism is then utilized to fix the integer values. Experimental results show that 100% success rate could be achieved for ultra-short baselines.

1. Introduction

The Global Navigation Satellite System (GNSS) is conceived as a viable alternative or complement to traditional attitude sensors for attitude determination of land, sea, air, and space vehicles [1–14]. The key to high-precision GNSS attitude determination is integer ambiguity resolution. Compared with motion-based ambiguity resolution methods [11,15–17] which exploit the time-varying receiver-satellite geometry, search-based methods [7,12–14,18–25] achieve instantaneous attitude determination and can be used in GNSS-challenged environments where frequent losses of lock occur. Among various search-based methods, the LAMBDA (Least-squares AMBiguity Decorrelation Adjustment) method [20,26] and its variants [6–9,21,22,27,28] have been widely used for their numerical efficiency and high success rate. The LAMBDA method requires both code and carrier phase observations. The contribution of code measurements is that they restrict the search space of integer ambiguities from infinity to a small region. A high success rate can benefit from a low code noise level. Conversely, a poor quality of code measurements results in degraded performance [8].

The present study seeks a solution to LAMBDA ambiguity resolution for attitude determination without aid of code measurements. This will be useful for applications under higher code noise and/or multipath environments (low-end GNSS receivers, urban canyons, etc.). The multivariate GNSS-attitude model [8,9,22] associates double-differenced (DD) phase ambiguities with multi-antenna platform attitude in a probabilistic manner. The scale of the probability space of ambiguities is related to baseline length. For ultra-short baseline attitude determination, the ambiguity probability space is sufficiently small. There is no need to use additional code measurements to restrict the ambiguity space. In this study, the Monte Carlo sampling (MCS) method [29–32] is employed to construct the probability density function (pdf) of ambiguities conditioned on DD phase observations. The constructed pdf is used to obtain the expectation and covariance of ambiguities, which are fed to standard LAMBDA for integer ambiguity resolution.

2. Quaternion-based GNSS-attitude model

Consider a set of m+1 ($m \geq 2$) receivers/antennas tracking n+1 common GNSS satellites. The doubled-differenced carrier phase observations formed with m independent baselines are cast into the following multivariate GNSS-attitude model [22].

$$\mathbf{\Phi} = \mathbf{GRF} + \lambda\mathbf{Z} + \mathbf{V}; \qquad \mathrm{Cov}(\mathrm{vec}(\mathbf{V})) = \mathbf{Q}$$
$$\mathbf{R} \in \mathbb{O}^{3\times3}; \qquad \mathbf{Z} \in \mathbb{Z}^{n\times m} \tag{1}$$

where $\boldsymbol{\Phi}$ is the ($n \times m$) matrix containing the DD phase observations from the m baselines, $\boldsymbol{F}$ is the ($3 \times m$) matrix of baseline coordinates (expressed in the local coordinate system of the multi-antenna platform), $\boldsymbol{R}$ is the (3×3) matrix of coordinate transformation from the local coordinate system to the GNSS reference system, $\boldsymbol{G}$ is the ($n \times 3$) matrix of double-differenced unit line-of-sight vectors, $\boldsymbol{Z}$ is the ($n \times m$) matrix of integer ambiguities (expressed in cycles), λ is the carrier phase wavelength, and $\boldsymbol{V}$ is the ($n \times m$) matrix of observation noises. The vector operator vec(·) is defined as stacking the columns of the ($n \times m$) matrix $\boldsymbol{V}$ into the vector vec($\boldsymbol{V}$) of order nm. $\boldsymbol{Q}$ is the covariance matrix of vec($\boldsymbol{V}$) and is constructed as follows

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$$\mathbf{Q} = \sigma^2 \begin{bmatrix} 1 & 0.5 & \cdots & 0.5 \\ 0.5 & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0.5 \\ 0.5 & \cdots & 0.5 & 1 \end{bmatrix}_{m\times m} \otimes \begin{bmatrix} 4 & 2 & \cdots & 2 \\ 2 & 4 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 2 \\ 2 & \cdots & 2 & 4 \end{bmatrix}_{n\times n} \tag{2}$$

where σ is the standard deviation of undifferenced phase noise and the operator $\otimes$ denotes the Kronecker product. The unknowns to be resolved in model (1) are the integer ambiguities and the 3 × 3 orthonormal attitude matrix $\boldsymbol{R}$.

In this study, the attitude matrix is parameterized with quaternion representation. Model (1) is further written as the following quaternion-based observation equation

$$\mathbf{\Phi} = \mathbf{G}\mathbf{R}(\mathbf{q})\mathbf{F} + \lambda\mathbf{Z} + \mathbf{V}; \qquad \mathrm{Cov}(\mathrm{vec}(\mathbf{V})) = \mathbf{Q}$$
$$\mathbf{q} \in \mathbb{R}^{4\times 1}; \qquad \mathbf{Z} \in \mathbb{Z}^{n\times m} \tag{3}$$

with

$$\mathbf{R}(\mathbf{q}) = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \tag{4}$$

where the quaternion $\mathbf{q} = (\mathbf{q}_0, \ q_4)^T$, $\mathbf{q}_0 = (q_1, \ q_2, \ q_3)^T$, and $\mathbf{q}^T\mathbf{q} = 1$.

3. Monte Carlo sampling approach

3.1. Constructing pdf of ambiguities

Based on model (3), the DD ambiguities are functions of attitude quaternion and DD phase observations

$$\mathbf{Z} = \frac{1}{\lambda}(\mathbf{\Phi} - \mathbf{G}\mathbf{R}(\mathbf{q})\mathbf{F} - \mathbf{V}) \tag{5}$$

where the quaternion $\boldsymbol{q}$ and observation noise matrix $\boldsymbol{V}$ are regarded as independent random variables.

The observation noises are assumed to be normally (Gaussian) distributed with mean zero and covariance $\boldsymbol{Q}$. The pdf of vec($\boldsymbol{V}$) is

$$p_{\mathrm{vec}(\mathbf{V})}(\mathbf{v}) = N(\mathbf{v}; \ \mathbf{0}, \ \mathbf{Q}) \triangleq \frac{1}{\sqrt{(2\pi)^{nm}\det\mathbf{Q}}} e^{-1/2\mathbf{v}^T\mathbf{Q}^{-1}\mathbf{v}} \tag{6}$$

The probability distribution of $\boldsymbol{q}$ can be inferred from a priori attitude information, which is usually obtained from coarse attitude sensors or dead reckoning. Given a priori quaternion with lower bound $\mathbf{q}^l$ and upper bound $\mathbf{q}^u$, a uniform distribution incorporating the norm 1 constraint can be formulated as follows

$$p_{\mathbf{q}}(\mathbf{q}) = U(\mathbf{q}; \ \mathbf{q}^l, \ \mathbf{q}^u) \triangleq \begin{cases} c & \mathbf{q}_0 \in [\mathbf{q}_0^l, \ \mathbf{q}_0^u], \quad \|\mathbf{q}_0\| \le 1, \quad q_4 = \pm\sqrt{1 - \|\mathbf{q}_0\|^2} \\ 0 & \text{elsewhere} \end{cases} \tag{7}$$

with

$$\int_{\substack{\mathbf{q}_0 \in [\mathbf{q}_0^l, \ \mathbf{q}_0^u], \\ \|\mathbf{q}_0\| \le 1}} c\,d\mathbf{q}_0 = 1 \tag{8}$$

where $\mathbf{q}_0^l$ and $\mathbf{q}_0^u$ are the vectorial parts of $\mathbf{q}^l$ and $\mathbf{q}^u$, respectively. The lower bound $\mathbf{q}^l$ and upper bound $\mathbf{q}^u$ can be set to the 3σ error bounds of a priori attitude estimation. In the absence of a priori attitude information, $\mathbf{q}_0^l$ and $\mathbf{q}_0^u$ can be set to $[-1, \ -1, \ -1]^T$ and $[1, \ 1, \ 1]^T$, respectively.

The pdf of ambiguities in terms of pdfs of $\boldsymbol{q}$ and vec($\boldsymbol{V}$) is [33]

$$p_{\mathrm{vec}(\mathbf{Z})}(\mathbf{z}) = \frac{\partial}{\partial \mathrm{vec}(\mathbf{Z})} \iint_{\mathbf{q},\mathbf{V}\in D_{\mathbf{z}}} p_{\mathbf{q}}(\mathbf{q}) p_{\mathrm{vec}(\mathbf{V})}(\mathbf{v}) \ d\mathbf{q} \ d\mathrm{vec}(\mathbf{V}) \tag{9}$$

with

$$D_{\mathbf{z}} = \left\{\mathbf{q}, \ \mathbf{V}; \ \frac{1}{\lambda}\mathrm{vec}(\mathbf{\Phi} - \mathbf{G}\mathbf{R}(\mathbf{q})\mathbf{F} - \mathbf{V}) \le \mathbf{z}\right\} \tag{10}$$

It is difficult to obtain the analytical expression of $p_{\mathrm{vec}(\mathbf{Z})}(\mathbf{z})$. The Monte Carlo method provides a convenient alternative to represent pdfs of random variables for nonlinear/non-Gaussian systems by a set of particles with associated weights.

The procedure of constructing $p_{\mathrm{vec}(\mathbf{Z})}(\mathbf{z})$ using the Monte Carlo method is given as follows.

First, draw N_s particles from $p_{\mathbf{q}}(\mathbf{q})$ and $p_{\mathrm{vec}(\mathbf{V})}(\mathbf{v})$

$$\mathbf{q}_i \sim p_{\mathbf{q}}(\mathbf{q}); \qquad \mathbf{v}_i \sim p_{\mathrm{vec}(\mathbf{V})}(\mathbf{v}) \tag{11}$$

Assign the particles with weights

$$w_i = \frac{1}{N_s}, \quad i = 1, \ 2, \ldots, \ N_s \tag{12}$$

Second, propagate the particles according to Eq. (5)

$$\mathbf{Z}_i = \frac{1}{\lambda}(\mathbf{\Phi} - \mathbf{G}\mathbf{R}(\mathbf{q}_i)\mathbf{F} - \mathbf{V}_i) \tag{13}$$

Finally, approximate the pdf $p_{\mathrm{vec}(\mathbf{Z})}(\mathbf{z})$ using the particles of ambiguities $\{\mathbf{z}_i, \ w_i\}_{i=1}^{N_s}$

$$p_{\mathrm{vec}(\mathbf{Z})}(\mathbf{z}) \approx \sum_{i=1}^{N_s} w_i \delta(\mathbf{z} - \mathbf{z}_i) \tag{14}$$

where $\delta(\cdot)$ is the Dirac delta function.

3.2. Ambiguity resolution and attitude determination

The expectation and covariance of ambiguities can be evaluated from the particles

$$\bar{\mathbf{z}} = \sum_{i=1}^{N_s} w_i \mathbf{z}_i \tag{15}$$

$$\mathbf{P} = \sum_{i=1}^{N_s} w_i (\mathbf{z}_i - \bar{\mathbf{z}})(\mathbf{z}_i - \bar{\mathbf{z}})^T \tag{16}$$

The expectation and covariance are then fed to LAMBDA for integer ambiguity resolution. The LAMBDA method utilizes Z-transformation to efficiently search for the 'best' integer vector which is closest to $\bar{\mathbf{z}}$ in the metric of $\boldsymbol{P}$ [26]. This criterion is generally valid for Gaussian distributed ambiguities. However, the pdf $p_{\mathrm{vec}(\mathbf{Z})}(\mathbf{z})$ in this study is non-Gaussian. The expectation does not contain much information about the correct ambiguities and only represents an average value evaluated on the probability space.

To address this problem, the LAMBDA algorithm implemented in this study does not return only one integer vector but a specific number of candidates in order of distance from $\bar{\mathbf{z}}$. A screening mechanism based on internal consistency checking is employed to select the candidate which best fits the GNSS-attitude model. The screening algorithm is presented as follows.

3.2.1. Attitude matrix extraction

For each integer ambiguity candidate $\hat{\mathbf{z}}_k, \ k = 1, \ 2, \ldots, \ N_c$, the corresponding attitude matrix $\hat{\mathbf{R}}_k$ can be extracted by solving the following linear matrix equation

$$\mathbf{\Phi} - \lambda\hat{\mathbf{Z}}_k = \mathbf{G}\hat{\mathbf{R}}_k\mathbf{F} \tag{17}$$

where $\hat{\mathbf{Z}}_k$ is the matrix form of $\hat{\mathbf{z}}_k$. For the two baseline case (m=2), the solution is a 3×2 matrix. The third column can be computed by the cross product of the first two columns.

3.2.2. Quaternion inversion

The quaternion candidate $\hat{\mathbf{q}}_k$ can be obtained from $\hat{\mathbf{R}}_k$ according to