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The optimisation of low-acceleration interstellar relativistic rocket trajectories using genetic algorithms

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ABSTRACT

A vast wealth of literature exists on the topic of rocket trajectory optimisation, particularly in the area of interplanetary trajectories due to its relevance today. Studies on optimising interstellar and intergalactic trajectories are usually performed in flat spacetime using an analytical approach, with very little focus on optimising interstellar trajectories in a general relativistic framework. This paper examines the use of low-acceleration rockets to reach galactic destinations in the least possible time, with a genetic algorithm being employed for the optimisation process. The fuel required for each journey was calculated for various types of propulsion systems to determine the viability of low-acceleration rockets to colonise the Milky Way. The results showed that to limit the amount of fuel carried on board, an antimatter propulsion system would likely be the minimum technological requirement to reach star systems tens of thousands of light years away. However, using a low-acceleration rocket would require several hundreds of thousands of years to reach these star systems, with minimal time dilation effects since maximum velocities only reached about $0.2c$. Such transit times are clearly impractical, and thus, any kind of colonisation using low acceleration rockets would be difficult. High accelerations, on the order of 1 g, are likely required to complete interstellar journeys within a reasonable time frame, though they may require prohibitively large amounts of fuel. So for now, it appears that humanity's ultimate goal of a galactic empire may only be possible at significantly higher accelerations, though the propulsion technology requirement for a journey that uses realistic amounts of fuel remains to be determined.

1. Introduction

A wealth of literature exists on optimising space trajectories, in particular interplanetary trajectories due to its application in the near-future. A majority of the research focuses on optimising trajectories for a specific propulsion system, rather than for a general propulsion system that utilises the rocket equation. Solar sails appear to be the favourite propulsion candidate for trajectory optimisation due to the fact that there is no fuel consumption, hence considerably simplifying the analysis: [1] used basic calculus to optimise the solar system exit speed for a spacecraft using a solar sail; [2] optimised interplanetary solar sail trajectories with respect to the flight time using particle swarm optimisation; [3,4] used evolutionary neurocontrol to optimise low-thrust interplanetary trajectories; [5] used sequential quadratic programming to optimise the flight time for a small spacecraft to reach the edge of the heliosphere using solar and nuclear electric propulsion systems; and [6] used a genetic algorithm to optimise the fuel consumption during orbital transfers. However, solar sails are not practical for interstellar travel since they require a constant external source of energy, which is not always present in the expanse of interstellar space. Research conducted in optimising interstellar trajectories have mostly been performed within a Newtonian model, thereby simplifying the analysis by ignoring the relativistic effects of time dilation.

The discovery that time is relative has raised many interesting discussions, and has produced a plethora of literature on its effect on interstellar travel. Within the scientific community, many authors have examined the effects of time dilation whilst travelling interstellar and intergalactic distances, though all but a few of the calculations were performed in flat spacetime. [7,8], and [9] considered the effect of an expanding universe when traversing intergalactic distances, and showed that a constant acceleration is necessary if one wishes to reach nearby galaxies within human lifetimes (though this is sensitive to the cosmological parameters used). In the currently favoured cosmological concordance model, a rocketeer accelerating at a constant rate of $g = 9.81$ ms^{-2} is able to reach 99% of the way to the edge of the universe well within a human lifetime ([9]), though upon return, many billions of years would have passed for those living on Earth.

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Optimising an interstellar trajectory is an extremely complex and difficult task, and producing the correct solution may not always be possible. Almost all attempts consider either a Newtonian or special relativistic approach, as a general relativistic approach compounds the difficulty of the task. [10] derived the optimality conditions for rocket trajectories in general relativity, though it was done from an analytical approach and did not consider any specific trajectories. To date, very little research has been performed on optimising interstellar trajectories in a general relativistic framework.

2. Theory

2.1. General theory of relativity

The assumption of the constancy of the speed of light c means that space and time could be unified into a single coordinate system where the position of a particle is $x^\alpha = (t, x, y, z)$. In the curved spacetimes of general relativity, a rocketeer will experience kinematic and gravitational time dilation. The proper time τ they experience is dependent on their velocity magnitude as well as the local spacetime geometry, described by a metric tensor $g_{\alpha\beta}$.

The equations of motion of a traveller in curved spacetime in Einstein summation convention is given by

$$\frac{d^2x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} + a^\alpha \tag{1}$$

where $a^\alpha = (a^t, a^x, a^y, a^z)$ is the four-acceleration (the relativistic analogue of three-acceleration), and the Christoffel symbols $\Gamma^\alpha_{\beta\gamma}$ describes the local spacetime geometry [11],

$$g_{\alpha\delta}\Gamma^\delta_{\beta\gamma} = \frac{1}{2}\left(\frac{\partial g_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial g_{\alpha\gamma}}{\partial x^\beta} - \frac{\partial g_{\beta\gamma}}{\partial x^\alpha}\right) \tag{2}$$

The parameters of a particle are related through two normalisation conditions:

$$g_{\alpha\beta}u^\alpha u^\beta = -c^2 \tag{3}$$

$$g_{\alpha\beta}u^\alpha a^\beta = 0 \tag{4}$$

where $u^\alpha = (u^t, u^x, u^y, u^z)$ is the four-velocity (the relativistic analogue of Newtonian three-velocity).

2.2. Milky Way mass model

To model the effect of spacetime curvature due to the mass of the Milky Way, the static weak field metric will be used, which describes the spacetime geometry in a weak, time-independent, gravitational field, such as that of the Milky Way ([12]). The static weak field depends on the Newtonian gravitational potential Φ, and is described by the metric

$$g_{\alpha\beta} = \mathrm{diag}\left(-c^2\left(1 + \frac{2\Phi}{c^2}\right), 1 - \frac{2\Phi}{c^2}, 1 - \frac{2\Phi}{c^2}, 1 - \frac{2\Phi}{c^2}\right) \tag{5}$$

The gravitational potential due to the Milky Way galaxy is made up from the gravitational effects of the bulge, disk, and dark matter halo. The Miyamoto-Nagai disk, Hernquist bulge, and Navarro-Frenk-White potential models are used to model the gravitational influence of the galactic disk, bulge, and halo, respectively.

The potential of the Miyamoto-Nagai disk ([13]) is given by

$$\Phi_d = -\frac{GM_d}{\sqrt{x^2 + y^2 + \left(r_d + \sqrt{z^2 + b_d^2}\right)^2}} \tag{6}$$

where $M_d = 10 \times 10^{10}M_\odot$ is the mass of the disk, r_d=6.5 kpc is the scale length of the disk, and b_d=0.26 kpc is the scale height of the disk ([14]).

The potential of the Hernquist Bulge ([15]) is given by

$$\Phi_b = -\frac{GM_b}{\sqrt{x^2 + y^2 + z^2} + r_b} \tag{7}$$

where $M_b = 3.4 \times 10^{10}M_\odot$ is the mass of the bulge and r_b=0.7 kpc is the scale length of the bulge.

The potential of the Navarro-Frenk-White Halo ([16]) is given by

$$\Phi_h = -\frac{GM_h}{\sqrt{x^2 + y^2 + z^2}}\ln\left(\frac{\sqrt{x^2 + y^2 + z^2}}{r_h} + 1\right) \tag{8}$$

where M_h is the mass of the halo and r_h is the scale length of the halo. The mass and scale lengths are calculated from the virial mass M_v of the halo, which is the enclosed halo mass at the virial radius R_v. The exact size of a galaxy is difficult to quantify as the halo mass density extends out continuously into intergalactic space, and the virial radius can be thought of as the radius beyond which the halo blends into the background matter in the universe. For the Milky Way, the virial mass is roughly $M_v = 150 \times 10^{10}M_\odot$ ([17]). The virial radius is calculated from the virial mass using ([18])

$$R_v = \left(\frac{2M_v G}{H_0^2\Omega_m\Delta_{th}}\right)^{1/3} \approx 294.5 \text{ kpc} \tag{9}$$

where $H_0 = 70.4 \times 10^{-3}$ kms^{-1} Mpc^{-1} is the Hubble constant, $\Omega_m = 0.3$, and $\Delta_{th} = 340$ is the Hubble constant, matter density of the universe, and over-density of dark matter compared to the average matter density, respectively [19].

The mass and scale lengths of the dark matter halo are related to the virial mass and virial radius via the dark matter halo concentration, which is described by the halo concentration parameter c_h, approximated by [20]

$$c_h \simeq 9.6\left(\frac{M_v}{10^{13}M_\odot}\right)^{-0.13}(1 + z)^{-1} \approx 12 \tag{10}$$

where z is the redshift, which is zero for host dark matter halos. The mass [21] and scale length [22] of the halo are then given by

$$M_h = \frac{M_v}{\ln(c_h + 1) - \frac{c_h}{c_h + 1}} \approx 91.4 \times 10^{10}M_\odot \tag{11}$$

$$r_h = \frac{R_v}{c_h} \approx 24.5 \text{ kpc} \tag{12}$$

The gravitational potential of the Milky Way is then the sum of each of the individual components.

2.3. Relativistic rocket

For a relativistic rocket, the proper acceleration a (i.e. the acceleration as experienced by the traveller) is related to the rate of change of mass of the rocket [23] by

$$\frac{1}{c}\int_0^\tau a\ d\tau = -\frac{v_e}{c}\int_0^\tau \frac{1}{m}\frac{dm}{d\tau}d\tau \tag{13}$$

where v_e is the effective exhaust velocity of the propellants. If the rocket expends all its fuel after a proper time of τ_f, then it is straightforward to show that

$$m_0 = m_r \exp\left(\frac{1}{v_e}\int_0^{\tau_f} a(\tau)d\tau\right) \tag{14}$$

where m_r is the final mass of the rocket. If the mass of the fuel is m_f, then $m_0 = m_f + m_r$, and hence

$$m_f = m_r\left[\exp\left(\frac{1}{v_e}\int_0^{\tau_f} a(\tau)d\tau\right) - 1\right] = m_r\beta \tag{15}$$

where β is the fuel-to-empty rocket mass ratio. Smaller values of v_e will result in a larger value of β, and hence more fuel will be required as