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A beacon configuration optimization method based on Fisher information for Mars atmospheric entry

Zeduan Zhao[a,b,c], Zhengshi Yu[a,b,c,*], Pingyuan Cui[a,b,c]

[a] Institute of Deep Space Exploration, Beijing Institute of Technology, Mail box 22, School of Aerospace Engineering, Beijing 100081, China
[b] Key Laboratory of Autonomous Navigation and Control for Deep Space Exploration, Ministry of Industry and Information Technology, Beijing 100081, China
[c] Key Laboratory of Dynamics and Control of Flight Vehicle, Ministry of Education, Beijing 100081, China

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ABSTRACT

The navigation capability of the proposed Mars network based entry navigation system is directly related to the beacon number and the relative configuration between the beacons and the entry vehicle. In this paper, a new beacon configuration optimization method is developed based on the Fisher information theory and this method is suitable for any number of visible beacons. The proposed method can be used for the navigation schemes based on range measurements provided by radio transceivers or other sensors for Mars entry. The observability of specific state is defined as its Fisher information based on the observation model. The overall navigation capability is improved by maximizing the minimum average Fisher information, even though the navigation system is not fully observed. In addition, when there is only one beacon capable of entry navigation and the observation information is relatively limited, the optimization method can be modulated to maximize the Fisher information of the specific state which may be preferred for the guidance and control system to improve its estimation accuracy. Finally, navigation scenarios consisted of 1–3 beacons are tested to validate the effectiveness of the developed optimization method. The extended Kalman filter (EKF) is employed to derive the state estimation error covariance. The results also show that the zero-Fisher information situation should be avoided, especially when the dynamic system is highly nonlinear and the states change dramatically.

1. Introduction

Driven by future Mars sample return requirements [1] and the science interest on more complex regions proposed by NASA [2,3], precision landing is pursued for the next generation of Mars missions. Until now, seven landers have successfully landed on the Mars surface. Curiosity has got the most accurate landing among the past missions, with the final landing point approximately 2 km from its target [4]. The active onboard guidance during the atmospheric entry phase which is firstly employed by the Mars Science Laboratory (MSL) mission is a main contributor to the improved landing accuracy and final landing altitude. However, it is still far away from the precision landing requirements. Due to the thick protective aeroshell, past landers only relied on the inertial measurement unit (IMU) to get the accelerations and angular rates information and employed the dead-reckoning technique to get the estimated states [5]. However, the initial knowledge error at entry interface and the drift error and biases of IMU will propagate along the entry trajectory without other measurement information. The entry guidance and parachute deploy trigger must rely on the limited estimated states and the parachute deploy ellipse will be no smaller than the navigation accuracy. To enhance the capability of the guidance, control and navigation system during Mars entry and reduce the parachute deploy ellipse, innovative navigation techniques are required to provide more information of the entry trajectory. Theoretically, if the range measurement between the entry vehicle and the beacon (whose position is known in advance) can be provided, the navigation performance can be dramatically improved, which is also the concept of the proposed Mars network based navigation [6–8]. Because radio measurement is easy to be implemented and has been preliminarily tested by the Curiosity, the radio measurement based navigation techniques are very prospective for the future Mars entry navigation [9–13].

In the Mars network based navigation scheme, the surface beacons are pre-deployed with known position knowledge before a specific Mars mission [14]. The Mars rovers or orbiters capable of radio navigation can also act as beacons. Radio navigation ability is directly relevant to the beacon numbers and their relative configuration with respect to the entry vehicle [15]. However, at present, only four orbiters (e.g. Mars

* Corresponding author at: Beijing Institute of Technology, Beijing 100081, China.
E-mail addresses: zhaozeduan@foxmail.com (Z. Zhao), yuzhengshi@gmail.com (Z. Yu), cuipy@bit.edu.cn (P. Cui).

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Odyssey, Mars Reconnaissance Orbiter, Mars Express, and the Mars Atmosphere and Volatile Evolution Mission) are capable of radio navigation [7,10,16–19]. The Indian Mangalyaan orbiter [20] is only equipped with low, medium and high gain antennas for communication with the Earth stations and may be not capable of spacecraft-to-spacecraft radio navigation. Furthermore, for the scale of the entry trajectory is rather small comparing to the size of Mars radius and Mars orbiter trajectory, the number of beacons visible for the whole trajectory can be very limited. Therefore, to make best use of the limited number of beacons to improve the navigation capability, the beacon configuration (positions of the beacons) should be optimized based on the specific mission and the corresponding entry trajectory.

Pastor et al. [14] have tried to determine the best configuration based on analyzing the navigation performance of several possible beacon positions with extended Kalman filter (EKF). Ely [21] utilized the genetic algorithm to minimize mean value of Mean Response Time, in order to improve the performance of the constellations, which is more efficient and systematic. To employ the optimization theory to get the best possible configuration scenarios, navigation capability related metrics should be modeled mathematically and included in the performance metrics and constraints. Observability analysis is a common method for navigation system analysis. Linearization and Lie algebra are commonly used for observability matrix derivation. The linearization method is computational relatively effective, but the observability of the linearized system cannot determine that of the original system definitely. Particularly, as the dynamic model of the entry vehicle is highly non-linear, the linearization may degrade the analyzing accuracy further. Lie algebra is based on the differential geometry theory. It is more accurate than the linearization method. However, Lie algebra is computational quite inefficient. Yu et al. [22] has compared the capability of the linearization method and Lie algebra on the background of Mars entry. The authors use the condition number of the observability matrix as the navigation performance index and employ the generic algorithm to get the optimized beacon configuration. However, the simulation results show the disability of the linearization method around the peak dynamic pressure region. Furthermore, the quadratic approximation is employed to ease the computational burden with Lie algebra and proved to be efficient.

Fisher information matrix (FIM) is another commonly used technique for observability analysis. According to the Cramér-Rao inequality, the estimation error covariance is no less than the inverse of FIM, and they are equal to each other only when the estimation method is an unbiased estimation [23]. Thus, FIM can be a measure of the observability of the navigation system and its related function can be included into the performance index for beacon configuration analysis and optimization. FIM has the feature that it only depends on the observation model and the characteristics of the observation noise. The dynamics of the system is not included in the information derivation. Thus, it is computationally effective and easy to get analytical forms for further deep and intuitive analysis. Yu et al. [15] have done a thorough FIM related observability analysis for Mars entry navigation, and derived the analytical relationship between the observability and the geometric configuration. However, the analysis is based on the assumption that at least three beacons are visible for the whole entry trajectory, which guarantees the observability of states. Situations with only one or two beacons are not studied in detail because the system is not fully observable. Observability related work based on FIM [24,25] also omits the 1 or 2 beacon cases. However, considering the limited number of radio beacons for Mars entry navigation, it is necessary to develop a configuration optimization method suitable for any number of beacons. Especially when beacons are less than three and the measurement information is further decreased.

Unlike FIM, which contains the information of the position or velocity vector, or full state vector, Fisher information (FI) in this paper is defined as the information of each state and derived analytically. Such FI is a scalar quantity, and does not require the navigation system to be fully observable. By maximizing the minimum average FI of each state, the navigation performance of the whole system can be improved. In addition, for navigation system with only one visible beacon, it may be a good choice to make fully use of the beacon to improve the estimation accuracy of a specific state which may be preferred for the guidance and control system. This can be achieved by modulating the developed optimization method to only maximize the FI corresponding to the specific state, at the cost of reducing the navigation performance on other states.

The rest of this paper is arranged as follows: Section II gives the dynamic model of the entry vehicle to determine the corresponding entry trajectory. The observation model of the navigation system is also constructed in this part. In Section III, the state related FI is derived with the maximum likelihood theory. Then the beacon position optimization method is developed considering the constraints of beacon visibility and maximum FI. Meanwhile, the optimization methods for both overall navigation performance and navigation performance of specific state are also introduced separately. This method can be used for the navigation schemes based on range measurements including, but not limited to, the Mars entry navigation based on radio measurements. In Section IV, configurations of different number of beacons are optimized using the proposed method. The resulting improvement of Fisher information matrix and navigation performance indicate the feasibility and necessity of the proposed beacon position optimization method for navigation design of Mars entry phase. Finally, main conclusions are given in Section V.

2. Construction of the radio navigation system

2.1. Determination of the Mars entry trajectory

The navigation performance of the radio navigation system is directly related to the relative configuration between the beacons and the entry vehicle. Hence, the beacon configuration determination should be associated with a specific entry trajectory. For the duration of Mars atmospheric entry phase is quite short, the rotation of Mars is omitted here. The 3-degree-of-freedom dynamic model of the entry vehicle in the Mars fixed Cartesian coordinate is established as follows [24]

$$\begin{bmatrix} \frac{d\boldsymbol{r}}{dt} \\ \frac{d\boldsymbol{v}}{dt} \end{bmatrix} = [\boldsymbol{v} \frac{1}{m}\sum \boldsymbol{F}_i] = \begin{bmatrix} \boldsymbol{v} \\ -g\boldsymbol{i}_r - \frac{D}{m}\boldsymbol{i}_v + \frac{L}{m}\cos\phi(\boldsymbol{i}_{v\times r}\times\boldsymbol{i}_v) + \frac{L}{m}\sin\phi\boldsymbol{i}_{v\times r} \end{bmatrix} \tag{1}$$

The state of the entry vehicle is defined by $\boldsymbol{X} = [x, y, z, v_x, v_y, v_z]^{\mathrm{T}}$. Meanwhile, $\boldsymbol{r} = (x, y, z)^{\mathrm{T}}$ and $\boldsymbol{v} = (v_x, v_y, v_z)^{\mathrm{T}}$ are the position and velocity vectors. ϕ is the bank angle, which is the only control variable during entry. m is the mass of the vehicle. L and D are the lift and drag force respectively. $\boldsymbol{i}_r = \frac{\boldsymbol{r}}{\|\boldsymbol{r}\|}$ is the unit vector from Mars center to the mass center of the vehicle, and $-g\boldsymbol{i}_r$ refers to the gravitational acceleration. $\boldsymbol{i}_v = \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ is the unit vector of velocity, and $-\frac{D}{m}\boldsymbol{i}_v$ is the drag acceleration. $\boldsymbol{i}_{v\times r} = \frac{\boldsymbol{v}\times\boldsymbol{r}}{\|\boldsymbol{v}\times\boldsymbol{r}\|}$ is the unit vector perpendicular to the $\boldsymbol{r}$-$\boldsymbol{v}$ plane, and $\frac{L}{m}\sin\phi\boldsymbol{i}_{v\times r}$ is the projection of the lift acceleration in the longitudinal plane. Similarly, $\frac{L}{m}\cos\phi(\boldsymbol{i}_{v\times r}\times\boldsymbol{i}_v)$ is the projection of the lift acceleration in the lateral direction. The magnitude of the gravitational acceleration, drag and lift can be calculated by

$$g = \frac{\mu}{r^2}, \quad D = \frac{1}{2}\rho V^2 Bm, \quad L = D(C_L/C_D) \tag{2}$$

where, μ is the Mars Gravity constant, ρ is the atmospheric density, B is the ballistic coefficient, V is the magnitude of $\boldsymbol{v}$, and C_L/C_D is the lift-to-drag ratio. An analytical exponential density model [9] is adopted here