Finite-time control for asteroid hovering and landing via terminal sliding-mode guidance

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\section*{Abstract}
This paper proposes a new nonlinear guidance algorithm applicable for asteroid both hovering and landing. With the new guidance, a spacecraft achieves its target position and velocity in finite-time without the requirement of reference trajectories. The global stability is proven for the controlled system. A parametric analysis is conducted to illustrate the parameters' effects on the guidance algorithm. Simulations of straight landing, hovering to hovering and landing with a prior hovering phase of the highly irregular asteroid 2063 Bacchus are presented and the effectiveness of the proposed method is demonstrated.

\section*{1. Introduction}

Space agencies around the world have shown great interest in asteroid exploration. Two spacecraft, NEAR shoemaker and Hayabusa, have landed on the surface of asteroids successfully \cite{1, 2, 3}. Hayabusa-2 for an asteroid sample mission was launched successfully in 2014 \cite{4}. In March 2015, the Dawn arrived at Ceres after exploration of Vesta \cite{5}. Several asteroid sample return missions such as MarcoPolo-R \cite{6} and OSIRIS-REx \cite{7} have also been proposed. To efficiently explore a target asteroid, flight close to the asteroid including body-fixed hovering and landing is beneficial and necessary. Body-fixed hovering helps to obtain high resolution measurements and also can simplify descent maneuvers for landing \cite{8}. Landing is essential to get the sample for sample return missions.

Open-loop control for hovering and soft landing has been studied. In the works of Yang et al. \cite{9} and Zeng et al. \cite{10}, a mass-rotating model \cite{11} is used to simplify the gravity of elongated asteroids. The relationship of the feasible hovering region with respect to the parameters of the gravity and magnitude of nominal control for spacecrafts propelled by low-thrust or solar-sail has been studied \cite{9, 12}. Zeng et al. \cite{12} also investigated effects of different types of nonideal solar sails on body-fixed hovering. Yang et al. \cite{13} designed a method to generate two impulse transfer orbits connecting two special hovering points (i.e. natural equilibrium points). For landing on the surface of an asteroid, Yang and Baoyin \cite{14} designed fuel-optimal control for descent trajectories by a homotopic method. Pinson and Lu \cite{15} developed a method through which the original fuel-optimal control problem is transformed into a second order convex problem. Herrera-Sucarrat et al. \cite{16} designed landing trajectories using manifolds and effects of the parameters of the asteroid's gravity on landing trajectories were studied.

For flight close to the asteroid, challenges arise because of model uncertainties and disturbances. Studies on closed-loop control for hovering or landing have also been pursued. Sawai et al. proposed a closed-loop control by which the position along one direction is fixed and find the linear stable regions for hovering \cite{8}. Broschart and Scheeres \cite{8} used dead-band control instead of the tight control in Ref. \cite{17} to make the hovering control more practical for application. In these studies \cite{8, 17}, the spacecraft is required to be at the hovering status initially otherwise it will not converge to the target hovering point. Guelman proposed a closed-loop guidance law by which the spacecraft can be controlled to reach the desired point and then maintain there under the condition that the hovering point is one of the equilibrium points \cite{18}. Hawkins et al. applied the ZEM/ZEV feedback guidance for soft landing on asteroids \cite{19}. However, the control methods proposed in Refs. \cite{8, 17, 19} cannot guarantee global stability for flight close to asteroids with uncertainties and disturbances.

Sliding-mode control (SMC) is an effective nonlinear control method by which stability can be guaranteed with bounded uncertainties and disturbances. Terminal sliding-mode control, as one type of SMC, has the advantage of fast and finite-time convergence compared...
with the conventional linear SMC [20]. Applications of terminal SMC include studies such as missile interception [21], robotic manipulators [22] and motor control [23]. For asteroid missions, Huang et al. [24] and Lan et al. [25] have applied linear and terminal sliding-mode control for tracking the reference descending trajectories, respectively. Lee et al. [26] proposed a continuous control method to control orbit and attitude simultaneously for hovering over an asteroid. A sliding-mode surface is defined on which the system will converge to the origin exponentially, but, the problem of stability with uncertainties is not discussed in the paper.

Recently, the second-order sliding-mode control [27,28] has been used in the control design of hovering and landing by Furfaro et al. [29,30]. The global stability in the presence of bounded uncertainties and disturbance is proved to be guaranteed. In Ref. [29], a new definition of sliding-mode surface is used to provide a virtual controller which is then tracked by an active control in the so-called multiple sliding-mode surfaces guidance. Using this method [29], the time required to reach the landing site can be predefined by the user and no reference trajectory is required. However, the controller is not applicable after the spacecraft arrives at the target point so that this method cannot be applied in hovering control. On the contrary, the spacecraft is able to maintain its position at the target point by the algorithm proposed in this current paper. In Furfaro’s paper on hovering control [30], a special terminal sliding-mode surface is defined and it can be reached in finite time with a bang-bang type of control. The spacecraft can be controlled to the hovering point and then maintain its position. But, the control is not continuous and the thrusters along each direction need to be switched multiple times. Different from Furfaro’s method [30], the control in this paper is piecewise continuous thus it is suitable for continuous-thrust propelled spacecraft. The generated piecewise continuous control can also be used to generate a bang-bang type of control through PWPF algorithms [29,31].

In this paper, a new terminal sliding-mode guidance is developed to obtain the desired piecewise continuous control which can be used for both hovering and landing. We use a general terminal sliding-mode surface [20] instead of the sliding-mode surface used by Furfaro et al. [29]. To develop the guidance algorithm, the relative position vector, which is the first sliding-mode surface vector in Ref. [29,30], is used to develop a new dynamical system. The components of the terminal sliding-mode surface vector are defined using the components of the relative position vector and relative velocity vector. Although the nonsingular control in [20] can be applied with the defined terminal sliding-mode surface, the time required for the transition between the initial and target points cannot be estimated analytically for flights close to asteroids. Different from the method in Ref. [20], in this paper the singular term in the control is set to be zero when the distance from the spacecraft to the target point is less than a predefined threshold. In this way, the control stays nonsingular and the sliding-mode surface vector maintains zero during the transfer to the target point once it becomes zero.

This paper makes three significant contributions: 1) the control is still effective after the spacecraft arrives at the target point. Therefore, the proposed terminal sliding-mode guidance can be used for both asteroid hovering and landing; 2) the control is proved to be finite time control and the time required to reach the sliding-mode surface can be predefined by the user and the upper bound of the total flight time can be analytical estimated; 3) the controlled system is proven to global stable with uncertainties and disturbances.

The rest of the paper is organized as follows. In Section II, dynamical equations of spacecraft’s flight close to an irregular asteroid are described and the control problem is formulated. In Section III, the terminal sliding-mode guidance algorithm is proposed and the properties of finite time and stability are proved. The methods for removing chattering and singularity are also presented in Section III. Properties and effects of the design parameters are analyzed in Section IV. In Section V, simulations of three different scenarios are presented to demonstrate the effectiveness of the proposed method. Section VI concludes this paper.

2. Problem formulation

2.1. Dynamical equations

Spacecraft’s controlled flight close to an asteroid is considered in this paper. To describe this motion, an asteroid’s body-fixed frame, as shown in Fig. 1, is used. Definition of this body-fixed frame $o-xyz$ is as follows: its origin is the mass center of the asteroid and its three axes are aligned with the three principle axes of inertia of the asteroid.

The dynamical equations are written as [14,15]

$$\begin{align*}
\dot{r} &= v \\
\dot{\mathbf{v}} &= -2\mathbf{\omega} \times \mathbf{v} - \mathbf{\omega} \times \mathbf{v} \times \mathbf{r} - \mathbf{v} \times \mathbf{\omega} \times \mathbf{r} + \nabla U(r) + d + a,
\end{align*}$$

(2)

where $\mathbf{r} = [x, y, z]^T$ and $\mathbf{v} = [u, v, w]^T$ are the position and velocity of the spacecraft, $\mathbf{\omega}$ is the angular velocity of the asteroid, $\nabla U(r)$ is the acceleration due to the gravity, $\mathbf{d} = [d_1, d_2, d_3]^T$ represents the uncertainties and disturbances, and $\mathbf{a}$ is the control provided by the thrusters of the spacecraft.

In this paper, the studied asteroids are assumed to be uniformly rotating, leading to $\mathbf{\omega} = 0$. Thus, Eq. (2) is simplified as

$$\dot{\mathbf{v}} = -2\mathbf{\omega} \times \mathbf{v} + \nabla U(r) + d + \mathbf{a},$$

(3)

For the convenience of describing the terminal sliding guidance, the scalar forms of the dynamical equations are used in this paper, leading to the following equations:

$$\begin{align*}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{z} &= v_z
\end{align*}$$

(4)

$$\begin{align*}
\dot{v}_x &= 2\omega v_y - \omega^2 r_x + \nabla U_x + d_1 + a_x \\
\dot{v}_y &= -2\omega v_x + \omega^2 r_y + \nabla U_y + d_2 + a_y \\
\dot{v}_z &= \nabla U_z + d_3 + a_z
\end{align*}$$

(7)

(8)

(9)

2.2. Control problem of flight close to asteroids

In this paper, two important types of close flight problems, which are landing and hovering, are considered. For both soft landing and ideal body-fixed hovering, the velocity at the final time $t_f$ should be zero, i.e. $\mathbf{v}_f = 0$. An important difference between these two kinds of flight problems is that the thrust will be turned off after arriving at the landing site while the thrust will continue working to maintain the position after spacecraft arrives at the new hovering point.

Assume a spacecraft is in body-fixed hovering over an asteroid initially with position $\mathbf{r}_0$ and velocity $\mathbf{v}_0$. We assume $\mathbf{v}_0$ is approximately zero. Denote the target position and velocity of the spacecraft at