Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Target detection in sea clutter via weighted averaging filter on the Riemannian manifold

Xiaoqiang Hua, Yongqiang Cheng, Yubo Li, Yifei Shi, Hongqiang Wang, Yuliang Qin

A R T I C L E I N F O

Article history:
Received 14 October 2016
Received in revised form 11 July 2017
Accepted 29 July 2017
Available online xxxx

Keywords:
Target detection
Riemannian manifold
Riemannian mean
Riemannian median
Weighted averaging filter

A B S T R A C T

This paper proposes a weighted averaging filter procedure combined with a Riemannian geometry method to carry out a target detection in sea clutter. In particular, the weighted averaging filter, conceived from a philosophy of the bilateral filtering in image denoising, is presented on a Riemannian manifold of Hermitian positive-definite matrix. This filter acts as a clutter suppression procedure in the detection framework of the algorithm proposed in this paper, and can improve the detection performance. The principle of detection is that if a location has enough dissimilarity from the Riemannian mean or median estimated by its neighboring locations, targets are supposed to appear at this location. Numerical experiments and real sea clutter data are given to demonstrate the effectiveness of the proposed target detection algorithm.

1. Introduction

The detection of targets embedded in sea clutter is an important subject in the field of radar signal processing. The environment of sea surface is very complicated and changeable. Sea clutter is a complex phenomenon influenced by environmental conditions, parameters of radar systems, and site configurations [1]. In a modern high-resolution radar system, sea clutter usually exhibits four properties of strong non-Gaussian, nonhomogeneous, non-stationary, and time-varying. Under those circumstances, it is difficult to achieve a satisfactory detection performance. Therefore, it is very necessary and meaningful for modern radars to improve their detection performance.

Few pulses can be used for target detection, since the dwell time of beam at target depending on the rotating speed of a scanning radar is very short. The classical fast Fourier transform (FFT) based constant false alarm rate (CFAR) detector [2] suffers from severe performance degradation owing to the poor Doppler resolution as well as the energy spread of the Doppler filter banks. To address those drawbacks, Barbaresco has done much work in the statistical geometry detection, and has proposed a generalized CFAR technique on a Riemannian manifold of Hermitian positive-definite (HPD) matrices, which was referred to as the matrix CFAR detector [3]. In the matrix CFAR detector, the radar received clutter data in each range cell in one coherent processing interval (CPI) is modeled as a HPD matrix. In addition, the Riemannian mean detector [3,4], and the median detector [5] are derived. Moreover, the Riemannian metric was deduced based on this parameterization. The existence and uniqueness of the mean and median had been proven in [6]. The matrix CFAR detector has been used for monitoring of wake vortex turbulences [7–9], and target detection in coastal X-band and HF surface wave radars [3]. It has been proven that the matrix CFAR detector has better detection performance than the classical FFT-CFAR detection algorithm [3,4]. Our previous research [10] has explored the matrix CFAR detector based on an alternative measure—the Kullback–Leibler divergence. The experimental results have also shown that the matrix CFAR detector has better detection performance than the classical FFT-CFAR algorithm. Furthermore, the performance of the Kullback Leibler-based matrix CFAR detector is better than that of the Riemannian distance-based matrix CFAR detector. The principle of detection is that targets are supposed to appear at a location if this location has enough dissimilarity from the Riemannian mean or median estimated by its neighboring locations.

Rather than exploring different distance measures used in the matrix CFAR detector, as our previous work [10], in this paper, we extend the framework of the matrix CFAR detector. We combine a weighted averaging filter, conceived from a philosophy of the bilateral filtering in image denoising [11], within the detection framework proposed by Barbaresco [3]. The purpose of the image denoising is to reduce the noise, and to enhance the image information. Similarly, we propose a weighted averaging filter to reduce the clutter, and to enhance the target signal. Concretely, the radar received clutter data in each cell in one CPI is modeled as a HPD matrix. The information of the target or clutter is represented by this HPD matrix. Then, the weighted averaging filter is imposed on these HPD matrices. The filtered HPD matrix in each

E-mail address: hxq712@yeah.net (X. Hua).

http://dx.doi.org/10.1016/j.ast.2017.07.042
1270-9638/© 2017 Published by Elsevier Masson SAS.

cell is a weighted average of HPD matrices of its surrounding cells. Finally, the detection decision is made based on these filtered matrices data. As that filter acts as a clutter suppression procedure, the detection performance can be improved.

The rest of this paper is organized as follows. In Section 2, we model the received signal using the HPD matrix. In Section 3, the Riemannian geometry of the space of HPD matrices is presented. The proposed detection algorithm is developed in Section 4. Then, we evaluate the performances of the proposed detection algorithm with different parameters as well as the matrix CFAR detector by simulated data and real sea clutter data in Section 5. Finally, conclusion is provided in Section 6.

1.1. Notation

A lot of notations are adopted as follows. We use math italic for scalars x, uppercase bold for matrices $\boldsymbol{A}$, and lowercase bold for vectors $\boldsymbol{x}$. The conjugate transpose operator is denoted by the symbol $(\cdot)^H$. $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ are the trace and the determinant of the square matrix argument, respectively. $\boldsymbol{I}$ denotes the identity matrix, and $\mathbb{C}(n)$ is the sets of n-dimensional vectors of complex numbers. The Frobenius norm of the matrix $\boldsymbol{A}$ is denoted by $\|\boldsymbol{A}\|_F$. For any $n \times n$ Hermitian matrix $\boldsymbol{A}$, $\boldsymbol{A} > 0$ means that $\boldsymbol{A}$ is a HPD matrix, and denoted by $\mathbb{P}(n)$. Finally, $\mathbb{E}(\cdot)$ denotes statistical expectation.

2. Signal modeled using HPD matrix

As mentioned above, the matrix CFAR detector can be illustrated in Fig. 1. The data $\boldsymbol{R}_i$ in the ith range cell is a HPD matrix estimated by the sample data $\boldsymbol{z}$ in the ith range cell in one CPI according to its correlation coefficient. Then, calculating the distance between the covariance matrix $\boldsymbol{R}_D$ of the cell under test and the mean matrix $\bar{\boldsymbol{R}}$ or median matrix $\hat{\boldsymbol{R}}$ of reference cells around the cell under test. Finally, the detection is made by comparing the distance between $\boldsymbol{R}_D$ and $\bar{\boldsymbol{R}}$ or $\hat{\boldsymbol{R}}$ with a given threshold γ. The Riemannian distance measure is used when calculating the distance as well as calculating the mean and median matrix, as the geometry of the manifold of HPD matrices is considered. In the following, we will give a brief description about how to construct the HPD covariance matrix from the sample data in each cell in one CPI.

For the radar received complex clutter data $\{\boldsymbol{z} = z_1, z_2, \ldots, z_n\}$ in each cell in one CPI, where n is the length of pulses, assuming $\boldsymbol{z}$ is a complex circular multivariate Gaussian distribution, $z \sim CN(\boldsymbol{0}, \boldsymbol{R})$, with zero mean and covariance matrix $\boldsymbol{R}$. The probability density function $\boldsymbol{p}(\boldsymbol{z}; \boldsymbol{0}, \boldsymbol{R})$ is given as follows [3],

$$\boldsymbol{p}(\boldsymbol{z}; \boldsymbol{0}, \boldsymbol{R}) = \frac{1}{\pi^n \det(\boldsymbol{R})} \exp\{-\boldsymbol{z}^H \boldsymbol{R}^{-1} \boldsymbol{z}\} \tag{1}$$

where π denotes the circumference ratio. The covariance matrix $\boldsymbol{R}$ is given by [3],

$$\boldsymbol{R} = \mathbb{E}[\boldsymbol{z}\boldsymbol{z}^H] = \begin{bmatrix} r_0 & \bar{r}_1 & \cdots & \bar{r}_{n-1} \\ r_1 & r_0 & \cdots & \bar{r}_{n-2} \\ \vdots & \ddots & \ddots & \vdots \\ r_{n-1} & \cdots & r_1 & r_0 \end{bmatrix}, \quad r_k = \mathbb{E}[z_i \bar{z}_{i+k}],$$
$$0 \le k \le n-1, 1 \le i \le n \tag{2}$$

where $r_k = \mathbb{E}[z_n \bar{z}_{n+k}]$ is called the correlation coefficient and $\bar{z}$ denotes the complex conjugate of z. $\boldsymbol{R}$ is a Toeplitz Hermitian positive-definite matrix with $\boldsymbol{R}^H = \boldsymbol{R}$. According to the ergodicity of a stationary Gaussian process, the correlation coefficient of data z can be calculated by averaging over time instead of its statistical expectation, as

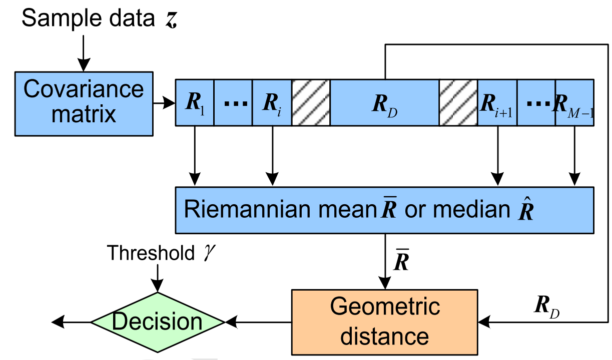

Fig. 1. Matrix CFAR detector [3].

$$\hat{r}_k = \frac{1}{n} \sum_{n=0}^{n-1-|k|} z(n)\bar{z}(n+k), \quad |k| \le n-1 \tag{3}$$

The pulse data in each cell in one CPI is modeled by Equation (1), and the information of target or clutter can be represented by its HPD matrix. Then, the new observation in each cell is a HPD matrix estimated by equations (2) and (3). These matrices of the range cells are embedded in $\mathbb{P}(n)$. According to the parameterization using the HPD matrix, the radar echo $\boldsymbol{z} = \{z_1, z_2, \ldots, z_n\}$ can be mapped into a n dimensional parameter space.

$$\boldsymbol{\psi}: \mathbb{C}(n) \to \mathbb{P}(n), \qquad \boldsymbol{z} \to \boldsymbol{R} \in \mathbb{P}(n) \tag{4}$$

Here $\mathbb{P}(n)$ forms a differentiable Riemannian manifold [12,13] with non-positive curvature [14,15]. HPD matrix manifold is a closed, self-dual convex cone, and serves as a canonical higher-rank symmetric space [16]. An excellent overview for HPD manifold is referred to [17,18].

3. Riemannian geometry of space of HPD matrices

In this Section, we overview some of basic mathematical knowledge related to this article, including the Riemannian distance, the geometric mean, and the geometric median. These are necessary for the detector design.

The space of HPD matrices $\mathbb{P}(n)$ is a differentiable manifold of dimension $n(n+1)/2$. The Riemannian distance between two different points $\boldsymbol{R}_1, \boldsymbol{R}_2$ on the manifold is defined by [19],

$$d_R^2(\boldsymbol{R}_1, \boldsymbol{R}_2) = \left\|\mathrm{logm}\left(\boldsymbol{R}_1^{-1/2} \boldsymbol{R}_2 \boldsymbol{R}_1^{-1/2}\right)\right\|_F^2 = \sum_{k=1}^{n} \log^2(\lambda_k) \tag{5}$$

where λ_k is the kth eigenvalue of $\boldsymbol{R}_1^{-1/2} \boldsymbol{R}_2 \boldsymbol{R}_1^{-1/2}$, $\mathrm{logm}(\cdot)$ is the logarithm map on the Riemannian manifold of HPD matrices.

Based on that distance metric, similar to algebraic mean and median, the definition of the Riemannian mean or median can be given as follows: Given a set of HPD covariance matrices $\{\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N\}$, the average matrix $\bar{\boldsymbol{R}}$ is the minimum value of the summation of p order distance between $\boldsymbol{R}$ and $\boldsymbol{R}_i$.

$$\bar{\boldsymbol{R}} = \arg\min_{\boldsymbol{R}} \frac{1}{N} \sum_{i=1}^{N} d^p(\boldsymbol{R}, \boldsymbol{R}_i) \tag{6}$$

where $d(\cdot,\cdot)$ denotes the Riemannian distance, and p is the order of the distance, when $p = 1$, $\bar{\boldsymbol{R}}$ denotes the median; when $p = 2$, $\bar{\boldsymbol{R}}$ is the mean.

The Riemannian mean associated with the Riemannian distance (5), of a set of N HPD matrices $\{\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N\}$, is given by [3],

$$\bar{\boldsymbol{R}}_{t+1} = \bar{\boldsymbol{R}}_t^{1/2} \,\mathrm{expm}\left\{-\varepsilon_t \left(\sum_{k=1}^{N} \mathrm{logm}\left(\bar{\boldsymbol{R}}_t^{-1/2} \boldsymbol{R}_k \bar{\boldsymbol{R}}_t^{-1/2}\right)\right)\right\} \bar{\boldsymbol{R}}_t^{1/2} \tag{7}$$

Please cite this article in press as: X. Hua et al., Target detection in sea clutter via weighted averaging filter on the Riemannian manifold, Aerosp. Sci. Technol. (2017), http://dx.doi.org/10.1016/j.ast.2017.07.042