Aerospace Science and Technology 70 (2017) 76–87

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Novel adaptive surrogate model based on LRPIM for probabilistic analysis of turbine disc

Jianxing Mao [a], Dianyin Hu [a,b,c,*], Da Li [a], Rongqiao Wang [a,b,c], Jun Song [d]

[a] School of Energy and Power Engineering, Beihang University, Beijing 100191, China
[b] Collaborative Innovation Center of Advanced Aero-Engine, Beijing 100191, China
[c] Beijing Key Laboratory of Aero-Engine Structure and Strength, Beijing 100191, China
[d] Mining and Materials Engineering, McGill University, Montreal, QC H3A 0C5, Canada

ARTICLE INFO

Article history:
Received 24 July 2016
Received in revised form 14 May 2017
Accepted 31 July 2017
Available online 7 August 2017

Keywords:
Probabilistic analysis
Surrogate model
Adaptive sampling method
Local radial point interpolation method (LRPIM)
Leave-one-out (LOO) validation test

ABSTRACT

Probabilistic lifetime assessment for aero-engine turbine disc is required to ensure structural safety and reliability. For probabilistic analysis of aero-engine turbine disc, a large amount of random variables involving load, geometry, and material properties result in a high dimensional nonlinear state function for the fatigue lifetime, which can become prohibitively expensive. This paper presents a novel adaptive surrogate model for the probabilistic analysis of an aero-engine turbine disc by integrating the local radial point interpolation method (LRPIM) and directional sampling technique. The directional sampling technique includes initial sampling, limit state recognition and subsequent sampling. In order to implement the high-dimension-probabilistic analysis for the turbine disc, an adaptive scheme is proposed involving three parts, i.e. scale adjustment of local support domain, convergence test and repeated procedure of subsequent sampling. Applied to an aero-engine turbine disc probabilistic analysis problem with 11 dimensional random variables, it is demonstrated that the novel approach proposed improves the accuracy and computational efficiency with reduced sampling amount as compared to other models such as response surface method (RSM), Kriging model (KM) and artificial neural network model (ANNM). A leave-one-out (LOO) validation test is performed to verify the robustness of the prediction of the adaptive surrogate model in the probabilistic analysis process of aero-engine turbine discs.

1. Introduction

Turbine disc, a critical safety component of an aero-engine, experiences significant tensile stresses due to centrifugal loading and thermal loading induced by the temperature gradient, which results in the dominant failure of low cycle fatigue (LCF). In addition, the failure is usually random in nature due to variation in turbine disc geometry, loading, material properties [1–4]. As a consequence, probabilistic life prediction is required to reduce failure risk and to ensure critical component's safety [5–8], i.e. the probability of failure P_f of a component based on a state function dependent on the vector of random input variables need to be determined. The probability of failure P_f normally can be determined using methods such as Monte Carlo simulation (MCS). However MCS can be computationally demanding as often a large number of nonlinear finite element (FE) analyses have to be performed to evaluate the state function. Such high computational expense of running complex and high fidelity simulations makes it practically difficult, especially when a large number of random variables are involved. To address this issue, approximation methods for probabilistic analysis to compute P_f at lower computational expense than MCS have been developed, such as the first-order reliability method (FORM) and second-order reliability method (SORM) [9]. These approximation methods require mathematical construction of the formulation between the objective parameter (e.g. fatigue life) and random variables for the concerned problem. However such construction can be rather challenging for complex engineering problem like the probabilistic analysis of turbine disc as explicit expressions are not always available. To avoid the derivation of mathematical formulation, a generalized strategy is to utilize surrogate models (also known as metamodels or approximation models) [10–13] based on so-called designs of experiments (DoE) [14,15] to approximately construct the state function with high computational accuracy and reduced computational cost.

Regarding the surrogate models, response surface model (RSM) [16,17], Kriging model (KM) [18,19], artificial neural networks model (ANNM) [20,21] have become the prevailing ones in engi-

* Corresponding author at: School of Energy and Power Engineering, Beihang University, Beijing 100191, China.
E-mail address: hdy@buaa.edu.cn (D. Hu).

http://dx.doi.org/10.1016/j.ast.2017.07.044
1270-9638/

neering design optimization. A series of comparison among these models in terms of accuracy and efficiency have been carried out [22–25]. In general, RSM requires the least of computational time, however, accuracy will significantly drop with increasing problem dimensions. KM and ANNM are always adopted as substitution for RSM in cases where the dimension is higher than 5. Recently, surrogate model established from radial basis function (RBF) has drawn considerable attention due to its advantages in accuracy [26], efficiency [27] and robustness [28]. RBF surrogate models have exhibited prominent ability to produce good fits for arbitrary contours on not only deterministic and stochastic [29], but also continuous and discrete [30] problems. Comparative studies have been conducted on RBF surrogate model with RSM, KM and some other models in high nonlinearity problems with dimensions from 2 to 10, where RBF surrogate model demonstrated outstanding accuracy and robustness [31]. However, when it comes to computationally expensive optimization problems, efforts are still required to improve the quality of samples distributed in variable space to obtain a capable surrogate model [32,33].

Besides surrogate models, there have also been considerable research efforts in DoE to generate samples possessing space-filling property (i.e. samples distribution as even as possible) as well as projective property (i.e. uniform samples in each dimension or the lower dimensional projective planes). Accordingly, several global sampling methods in design space such as full-factorial design [34], orthogonal array design [35], central-composite design [36], and Latin hypercube design (LHS) [37] were often used. Among these sampling methods, LHS has proven to be the most capable tool due to its excellent space-filling and projective properties [38].

The above-mentioned methods are widely applied in engineering problems with low-order nonlinear problems in low-dimensional space. Nevertheless, the application of current surrogate models and sampling methods to a high-dimension problem for the probabilistic analysis of the aero-engine turbine disc remains challenging, because of the following reasons:

(1) Global sampling technique cannot effectively recognize the nonlinear limit states, which results in larger amount of samples and reducing the efficiency of surrogate models. Thus, a more efficient sampling method needs to be developed to achieve the limit state recognition.

(2) A significant drop in computational accuracy will be encountered when the dimension for random variables is higher than 10, even if the sample amount continuously increases. Hence, a surrogate model with satisfactory accuracy and good robustness needs to be constructed, aiming at a feasible solution to the probabilistic analysis on a turbine disc for high-dimension problem.

(3) It is always difficult to achieve an excellent match between sampling method and surrogate model, meaning more efforts should be spent on. Therefore, an adaptive scheme for the combination of sampling method and surrogate model is required in order to make the constructed model accurate, efficient and robust.

In light of the above, the present study focuses on the development of a surrogate model introducing the local radial point interpolation method (LRPIM) to construct a high-dimension-nonlinear state function. The scale of local support domain can automatically adjust to avoid the ill-condition of coefficient matrix. Accordingly, an adaptive scheme was proposed, involving a directional sampling (DS) process to identify limit states in random variable space and a subsequent samples generator to realize uniformity with optimization of potential energy. Finally the numerical application on the probabilistic analysis of the turbine disc was performed to validate the method.

This paper is organized as follows: first, the development of an adaptive scheme by improving the computational accuracy and expense is presented in Section 2, involving a developed LRPIM-based surrogate model in details as well as a DS method. Then probabilistic analysis on the turbine disc of an aero-engine is then performed and discussed in Section 3. In the end, some summary remarks of this study are presented in Section 4.

2. Adaptive scheme and model construction

Fig. 1 illustrates the framework implemented for the probabilistic analysis. The proposed adaptive scheme includes three components, directional samples generation, LPRIM formulation, and probabilistic analysis using LRPIM-based surrogate model, as outlined in Fig. 1. They are briefly introduced in the following.

(1) Generate directional samples

Initial n_1 samples are generated to achieve some basic information of the concerned random variable space. Meanwhile, n_2 samples are generated using golden section search (GSS) method. By this step, nonlinear limit state with multiple failure boundaries will be identified based on the information provided by the $n_1 + n_2$ samples. Then, subsequent n_3 samples utilizing Min-SW procedure are generated. The aim of these samples is to gather additional information from a global perspective. Thus, samples tend to be uniform so as to avoid any possible omission but without overlapped or closely located nodes to improve the quality of coefficient matrix in LRPIM. These samples are called sampling nodes, which serve as the database for the LRPIM formulation. A procedure of normalization with total samples n (i.e. $n = n_1 + n_2 + n_3$) is required to eliminate the difference of magnitude order among random variables. The detailed process is described below in Section 2.4.

(2) Formulate LRPIM

During the formulation stage of surrogate model, the coefficient matrix in LRPIM is constructed with a specific type of radial basis function (RBF). Then a convergence test is performed. If the convergence test on the subsequent samples returns true, the samples to calculate the trial function matrix will be determined. Otherwise, another set of subsequent samples (6 samples in this study) should be added. The surrogate model is then reconstructed until the convergence test passes.

(3) Implement probabilistic analysis

Afterwards, n_{MC} samples are generated through MCS in order to perform probabilistic analysis, following the LRPIM formulation. These samples are called calculating points. Then the state function value is computed on each calculating point by the established LRPIM surrogate model. Consequently, the computational expense is effectively reduced since all of the samples are obtained from the surrogate model instead of the FE simulations. Based on the MCS results, reliability or failure probability is calculated. Eventually, error analysis and LOO test are conducted to evaluate the accuracy, efficiency and robustness of the proposed model.

2.1. Radial point interpolation method

Mathematically, interpolation is a numerical method of obtaining new data points within a domain fulfilled by a discrete set of known data points. This discrete set can be obtained via sampling or experiments. Point interpolation method (PIM) is based on scattered nodes within the concerned domain, using polynomial basis functions [39,40]. The general obstacle of PIM is singular moment matrix, which if occurs, will lead to the PIM process breaking down. In order to solve this problem, nonsingular moment matrix need to be created. In this study, radial point interpolation method (RPIM) developed by Wang and Liu [41,42], combining PIM with radial basis functions (RBFs), is employed.

A standard RPIM formulation is described as

$$u^h(\mathbf{x}) = \sum_{i=1}^{n} R_i(\mathbf{x}) a_i = \mathbf{R}^T(\mathbf{x})\mathbf{a} \tag{1}$$