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A modified elliptic integral method and its application in three-dimensional honeycombs

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ABSTRACT

In this paper, a modified elliptic integral method for the geometrically nonlinear analysis of Timoshenko beam is developed. Based on the elliptic integral method constructed by Sinclair, the proposed method also considers the axial, shearing and bending deformations of the beam, which makes up the deficiency of Sinclair's method without considering the shearing deformation at the fixed-end. In the geometrically nonlinear analysis of the three-dimensional honeycombs, the proposed method can be used to describe the unit cell as the elliptic integral equations, and the closed nonlinear equations can be obtained and solved by Newton iteration method. The numerical examples and three-point bending experiment indicate that, compared with Sinclair's method, the results obtained by the proposed method are in good agreement with those of the numerical simulation and experiment, especially for the beam with larger ratio of height to length. Therefore, the modified elliptic integral method developed in this paper has higher accuracy and wider range of application in the analysis of honeycombs.

1. Introduction

The elliptic integral method (EIM) is an effective method to study the geometrically nonlinear beam, which is used by Timoshenko in the study of buckling curve of the Euler pole [1]. Subsequently, researchers have promoted EIM. Drucker et al. used EIM to study the situation that the horizontal cantilever beam was subjected to vertical load at the free-end [2]. Nallathambi et al. further studied the large deflection of cantilever beam under follower load at the tip [3–5]. However, these methods used the Euler beam theory, which did not consider the effects of axial and shearing deformations of the beam. In 1978, Sinclair improved EIM in the study of stability of the beam [6]. Since the axial, bending and shearing deformations of the beam were all considered, the more accurate results were obtained. However, Sinclair ignored the shearing deformation at the fixed-end, which led to some error in the computational results.

With the innovation of industrial technology, especially the development of three-dimensional (3D) printing technology, the 3D negative Poisson's ratio (NPR) materials have attracted the attention of many researchers in recent years. NPR material, also known as auxetic material, gets fatter perpendicular to the applied force when it is stretched, or becomes smaller when it is compressed. NPR effects of most auxetic materials can be achieved through the geometric changes of microstructures. According to the deformation mechanism of NPR effects, the 3D auxetic materials can be divided into two kinds of structural forms, namely bending controlled [7–17] and twisting controlled [18–20] materials.

The experiment, finite element method (FEM) and analytical method are mainly used to study the auxetic materials. In the analytical method, the auxetic material is generally assumed as a macroscopic homogeneous structure. Then, the mechanical analysis for the unit cell is carried out by taking advantage of the periodicity of the structure, and the macroscopic equivalent parameters are obtained. For the analytical analysis of 3D auxetic materials, the more common methods are displacement method, energy method and so on. It should be noticed that the theoretical analysis of the mechanical properties of the auxetic materials mainly focused on the small deformation problems, and the researches on geometric nonlinearity are rare. EIM is a common method in the geometrically nonlinear analysis of cellular materials. Zhu et al. applied EIM to the geometrically nonlinear analysis of honeycombs and gave the equivalent stress–strain relation [21], which provides us with a guiding idea. After that, Qiu et al. improved EIM by considering the axial deformation in the study of honeycombs [22]. Yang et al. modified EIM under the assumption that the shearing force is constant in the study of the 3D auxetic materials [8].

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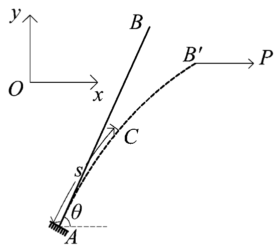

Fig. 1. An inclined cantilever beam subjected to horizontal concentrated load.

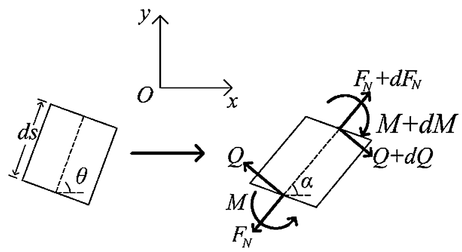

Fig. 2. Deformation diagram at fixed-end of the beam.

Based on the study of Sinclair, a more reasonable description of the shearing angle at the ends of the beam is given to improve the work of Ref. [6] and the computational accuracy of EIM. Meanwhile, the solution of geometrically nonlinear cantilever beam with inclination subjected to concentrated load is deduced. Furthermore, the modified elliptic integral method is applied to the 3D auxetic material of Ref. [7], and the parameter curves of the material considering geometric nonlinearity are obtained.

2. The cantilever beam subjected to concentrated load

2.1. The elliptic integral method proposed by Sinclair

EIM is an analytical method to study the geometric nonlinearity of the beam, which is applicable to the cantilever beam subjected to concentrated load at the free-end. The initial length, cross-sectional area and moment of inertia of the cantilever beam AB with the horizontal inclination $\theta \in [0, 2\pi]$ (positive in the counter-clockwise direction) depicted in Fig. 1 are l, A and I, respectively. Young's modulus and Poisson's ratio of the material used are E and ν, respectively, and the shearing correction factor is κ. The flexural deformation of the representative beam subjected to horizontal concentrated load P at free-end B is also shown by dotted line in Fig. 1. Assuming that the included angles between the axis of the beam and the horizontal line at the fixed-end and free-end are α and β, respectively. Then, the horizontal load P, the horizontal and vertical projected lengths x and y of deformed beam all can be expressed as functions of α and β in the elliptic integral form. Based on Timoshenko beam theory, Sinclair considered the axial, shearing and bending deformations and the corresponding formulas according to the model of Fig. 1 are as follows

$$P = \frac{EI}{l^2}\lambda \tag{1}$$

$$x = \frac{l}{\sqrt{\lambda}}\left[\Theta_1 - 2\Theta_2 - \frac{\delta k}{2}\left(-\cos^2\frac{\beta}{2}\cdot\Theta_1 + \cos\beta\cdot\Theta_2 + \cos^2\frac{\beta}{2}\sin\frac{\alpha}{2}\sin 2\eta\right)\right] \tag{2}$$

$$y = \frac{2l}{\sqrt{\lambda}}\cos\frac{\beta}{2}\cos\eta\left[1 + \frac{\delta k}{4}(\cos\alpha + \cos\beta)\right] \tag{3}$$

in which

$$\delta = \frac{\lambda I}{Al^2} \tag{4}$$

$$\sqrt{\lambda} = \Theta_1 - \frac{\lambda I}{Al^2}\left[\frac{k}{4}\cos\beta\cdot\Theta_1 + \left(1 - \frac{3k}{4}\right)(\Theta_1 - 2\Theta_2)\right] \tag{5}$$

$$\Theta_1 = K\left(\cos\frac{\beta}{2}\right) - F\left(\eta, \cos\frac{\beta}{2}\right) \tag{6}$$

$$\Theta_2 = E\left(\cos\frac{\beta}{2}\right) - E\left(\eta, \cos\frac{\beta}{2}\right) \tag{7}$$

$$\eta = \arcsin\frac{\cos(\alpha/2)}{\cos(\beta/2)} \tag{8}$$

$$k = \frac{2(1+\nu)}{\kappa} \tag{9}$$

It should be noted that the inclination of the cantilever beam defined in Fig. 1 differs from that defined in the Ref. [6]. Since it is necessary to calculate the square roots several times in the process of deriving the analytical formulas, the analytical formulas may have sign difference in different quadrants. After verification, the model chosen in this paper and the above analytical formulas all can be used when $0 < \theta < 2\pi$, which will greatly facilitate the subsequent application.

Sinclair made $\alpha = \theta$, which ignored shearing deformation of the beam at fixed-end and assumed that the inclination of the axis at the fixed-end is the same as the initial inclination of the beam. Then, Eqs. (1)–(3) all contain only one independent variable β by substituting Eqs. (4)–(9). If the horizontal load P is given, β can be obtained by the Newton iteration method, x and y are obtained further, and vice versa.

2.2. Modified elliptic integral method

It should be noted that there is a certain error in the approximation of Sinclair. Considering the shearing deformation of the beam at the fixed-end and adding an iterative parameter in this paper, the analytical results are closer to the true values.

Considering Eq. (1), Eq. (4) can be expressed as

$$\delta = \frac{P}{EA} \tag{10}$$

From Fig. 2, the shearing angle can be obtained as follows

$$\gamma = \frac{P\sin\alpha}{\kappa GA} = k\delta\sin\alpha \tag{11}$$

It can be seen that δ is a small quantity from Eq. (10).

As shown in Fig. 2, α is equal to the initial inclination θ of the beam minus the shearing angle γ. Considering Eq. (11), α is given by

$$\begin{aligned}\alpha &= \theta - \gamma = \theta - \delta k\sin\alpha \\ &= \theta - \delta k\sin\theta\cos(\delta k\sin\alpha) + \delta k\cos\theta\sin(\delta k\sin\alpha) \\ &= \theta - \delta k\sin\theta + O\left(\delta^2\right)\end{aligned} \tag{12}$$

Ignoring the second-order small quantity, we have

$$\alpha = \theta - \delta k\sin\theta \tag{13}$$

Note that the right side of Eq. (13) contains δ, which means that the cyclic assignment occurs between Eq. (13) and Eqs. (4)–(8). To avoid this problem, δ is regarded as an independent parameter and an equation associated with δ is complemented from Eq. (10). Then, Eqs. (1)–(3) all contain two independent variables δ and β by substituting Eqs. (5)–(9) and Eq. (13). Simultaneously, θ is formally regarded as a variable in order to facilitate the rotation of the coordinates in the following. Therefore, the right sides of Eqs. (1)–(3) are denoted as $P(\theta, \beta, \delta)$, $x(\theta, \beta, \delta)$ and $y(\theta, \beta, \delta)$, respectively.