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Analytical modelling for thermo-mechanical analysis of cross-ply and angle-ply laminated composite plates

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A B S T R A C T

In the present work, the thermo-mechanical response characteristics of cross-ply as well as angle-ply laminated composite plates in closed form are investigated. The laminated composite plates are modelled in an axiomatic framework based on an inverse hyperbolic shear deformation theory (IHSDT). The governing differential equations are yielded by implementing the principle of virtual work. These governing equations are solved separately for cross-ply and angle-ply plates by consideration of stiffness characteristics of these plates. The sinusoidal and uniform transverse mechanical loading is considered with the linear and non-linear distribution of temperature across the depth of the plate. The simply supported composite plates are considered and Navier solutions satisfying the associated boundary constraints are developed. The mechanical and thermo-mechanical response characteristics of cross-ply and angle-ply plates under such conditions are investigated and the results are presented in the non-dimensional form. A comparison of the obtained results with the existing results for a variety of examples is presented and based on this comparison study, the validity, applicability and accuracy of IHSDT for such analysis is ensured. The applicability and accuracy of IHSDT for thermo-mechanical responses of laminated composite plates is ascertained on the basis of results presented. The effects of span thickness ratio, lamination sequence, loading and thermal conditions, material anisotropy ratio, and aspect ratio on the thermo-mechanical response characteristics are also concluded.

1. Introduction

In the present era, there is a vast utilisation of laminated composite structures due to their enhanced mechanical performance, high specific strength and stiffness, resistance to corrosion, inherent surface finish, impact strength, ease of fabrication, chemical resistance and improved thermal properties. This leads to wide application of these advanced materials in many engineering applications such as automobile, marine structures, sports equipments, aerospace vehicle design, nuclear power plants, etc. The increased use of composites at elevated temperature environment have encouraged the researchers to predict the behaviour of composites under the thermal loading in order to optimise the usage of composites and ensure a superior design.

The composite structures have been investigated through many approaches in the past. The complicating effects of these composite structures have been addressed in a variety of ways. One such requirement is the existence of shear deformation effects in the composite materials. The effect of shear deformation was illustrated by Reissner [1] and Mindlin [2] for static behaviour of composite plates by implementing first order shear deformation theory (FSDT). However, FSDT requires a factor for the shear correction which is dependent on lamination sequence, boundary conditions, etc. [3]. A variety of theories were developed during last three decades. The development of these theories is illustrated extensively in the review articles [4–10]. In the displacement based axiomatic framework, the development of theories is observed in the equivalent single layer (ESL) approach, layer wise (LW) approach, and zig-zag (ZZ) approach. In the framework of ESL theories, the development is mainly focused on the shear deformation and the theories are termed as higher order shear deformation theories (HSDTs). The HSDTs can be expressed in polynomial or non-polynomial function of thickness coordinates. In polynomial shear deformation theory (PSDT), the displacement field is specified with Taylor series expansion of thickness coordinates. The significant work in the development of PSDTs is due to Reddy [11], Kant and Pandya [12], Maiti and Sinha [13], Ferreira et al. [14], Kant and Swaminathan [15], Talha and Singh [16], etc. However, a non-polynomial function of thickness coordinate is used in non-polynomial shear deformation theory (NPSDT). The various

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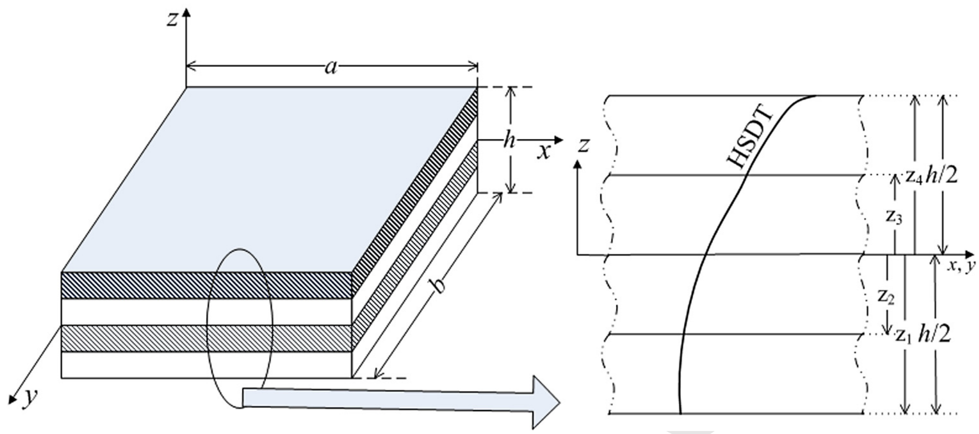

Fig. 1. Laminate geometry and co-ordinate system.

non-polynomial functions for instance trigonometric [17–21], exponential [22–24], hyperbolic [25], inverse trigonometric [26,20] and inverse hyperbolic [27] functions had been developed. The three-dimensional elasticity solutions for rectangular composite plates were presented by Pagano [28]. Savoia and Reddy [29] presented three-dimensional elasticity solution for multilayered plates using variation approach. Denmasi [30] presented three dimensional analytical solution for isotropic plates. Chaudhuri and Kabir [31] presented closed form solution for clamped laminated plates in the framework of first order shear deformation theory. Vel and Batra [32] implemented closed form solution in terms of infinite series to examine the effect of boundary conditions on three dimensional deformations of multilayered plates. Piskunav et al. [33] developed higher order rational transverse shear deformation theory for static analysis of anisotropic plates and shells using analytical solution for both cross-ply and angle-ply plates and shells. Oktem and Chaudhuri [34,35] employed Levy type closed form solution in order to study the effect of boundary conditions on deflection of cross-ply plates. Grover et al. [36] developed finite element formulation and examined the effects of boundary conditions on static, buckling and free vibration response of laminated plates.

The analyses of composite structures under the thermal environment have been investigated by various researchers using various displacement based theories. Wu and Tauchert [37] investigated the behaviour of cross-ply and angle-ply rectangular laminates with simply supported conditions using classical laminated plate theory (CLPT) under constant and linearly varying temperature field. Reddy and Hsu [38] presented analytical solutions for the cross-ply plates under thermal bending and used finite element formulation to illustrate results for angle-ply laminates. Other finite element models were also presented for thermal buckling, static and dynamic analysis [39,40]. Jin and Yao [41] examined the hygrothermal behaviour of laminated composite plates in terms of an improved C^0-type global–local model. Khdeir and Reddy [42] implemented Levy solution to investigate thermo static response of cross-ply composite plate under various boundary constraint using CLPT, FSDT and HSDT. Bhaskar et al. [43] investigated thermo-elastic behaviour of laminated plates and strips using the equilibrium conditions. Bektas and Sayman [44] analysed symtric cross-ply and angle-ply plates under constant thermal field across the thickness. Zenkour [45] presented Navier type closed form solution for cross-ply using sinusoidal plate theory (SPT) [17]. Li et al. [46] investigated the thermo-mechanical bending behaviour of functionally graded sandwich plates using a refined plate theory. Mechab et al. [47] studied thick orthotropic and cross-ply plates under thermal gradient using various higher order theories including HSDT and SPT using an analytical solution. Han et al. [48] presented an enhanced FSDT including the transverse normal effect for thermo-mechanical analysis of laminated composite plates.

It is observed from the literature that the analytical solutions are developed only for the cross-ply plates in the framework of recently developed NPSDTs. The analytical solutions for theories developed in the recent past are not available for the angle-ply plates. Also, the analytical solutions for the thermo-mechanical response characteristics of cross-ply plates are available. However, the solutions regarding angle-ply laminates are very rare in the literature. In the present work, a recently proposed inverse hyperbolic shear deformation theory (IHSDT) is employed to model the composite plates. The IHSDT has proved its accuracy by providing the analytical solutions for cross-ply plates and finite element solutions for cross-ply and angle-ply plates. However, its efficacy is not ensured for thermo-mechanical response characteristics in the existing literature. Additionally, the analytical solutions for angle-ply plates using IHSDT are not reported in the literature. Therefore, the objective of present work is to present analytical solutions for thermo-mechanical behaviour of cross-ply as well as angle-ply plates. The effect of lamination sequence, span-thickness ratio, material anisotropy, loading conditions, linearly varying and non-linearly varying temperature field, aspect ratio are examined on the thermo-mechanical responses.

2. Mathematical formulation

A fiber-reinforced layered composite plate having length a, breadth b and overall thickness, h subjected to a static transverse load $q(x, y)$ and a temperature field $T(x, y, z)$ is considered in the present work. The plate consists of n number of equal thickness layers stacked together to form the laminated plate. The schematic of a laminated composite plate in the Cartesian co-ordinate system is shown in Fig. 1.

2.1. Displacement and temperature field

The present formulation is based on the axiomatic formulation in which the displacements are primary variables. The in-plane displacements (u, v) and transverse displacement (w), at any point (x, y, z) in the laminated composite plate are expressed in the framework of an inverse hyperbolic shear deformation theory (IHSDT) [27].

$$u(x, y, z) = u_0(x, y) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x \tag{1a}$$

$$v(x, y, z) = v_0(x, y) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y \tag{1b}$$

$$w(x, y, z) = w_0(x, y) \tag{1c}$$