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# N-objective genetic algorithm to obtain accurate equivalent single layer models with layerwise capabilities for challenging sandwich plates ☆

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ABSTRACT

This paper presents refined equivalent single layer plate theories develop by an effective N-objective optimization method, considering multiple displacements and stresses as output parameters. The refined plate theories reported belong to Best Theory Diagrams (BTDs), in which the minimum number of terms that have to be used to achieve a desired accuracy can be read. Maclaurin, high order zig-zag, trigonometric, exponential and hyperbolic terms are employed in order to investigate their influence on several static mechanical studies for sandwich plates. The used refined models are develop via the Unified Formulation developed by Carrera. The governing equations are derived from the Principle of Virtual Displacement (PVD), and Navier-type closed form solutions have been obtained in the case of simply supported plates subjected to bisinuisoidal transverse pressure. BTDs have been constructed using the Axiomatic/Asymptotic Method (AAM) and genetic algorithms (GA). The results are compared with the layer-wise solution in several benchmarks proposed in the literature. It is shown that the ESL plate models can accurately describe the displacement field and the mechanical stress fields predicted by a LW model with less computational effort, even for extreme theoretical cases. Furthermore, the method presented allows the user to analyze the influence of numerous test functions in a single run. The combined use of CUF, AAM and GA is a powerful tool to evaluate the accuracy of any structural theory.


## 1. Introduction

Sandwich plates are multilayered structures consisting of one or more high-strength, stiff layers (faces), bonded to one or more low-density, flexible layers (core). This configuration gives the sandwich material system high stiffness-to-weight ratio and high-energy absorption capability related to the application of sandwich structures in the construction of civil and military aircrafts, marine structures and turbine blades [1–3].

A considerable amount of literature have been published on the modeling, analysis and design of sandwich structures. These include reviews by Habip [4], Noor et al. [5], Reddy and Robins [6], Librescu and Hause [7], Vinson [8], Carrera [9–11], Carrera and Brischetto [12]. An excellent review on plate and shell theories was recently provided by Caliri et al. [13], in which a vast number of approaches to the plate/shell problem available in the literature was presented.

A unique and exhaustive classification of 2D models for layered/sandwich structures is difficult to make. If the theories are classified according to the variable description in the layers, two different approach can be distinguished: the Equivalent Single Layer (ESL) and the Layer-Wise (LW) models. Excellent reviews of existing ESL and LW models can be found in Refs. [5,6,14–16]. According to the ESL, a plate/shell model can be analyzed considering it as a single equivalent lamina. In the LW approach, the displacement field is defined independently in each layer and the continuity at the interfaces is imposed. Theories based on the LW approach have a quasi-three dimensional predictive capabilities; however, the computational effort is excessively high for most practical applications. On the other hand, ESL models reduced the computational cost but often fail to model the zig-zag shaped cross-sectional distortion typical in heterogeneous laminates. In order to increment the accuracy of ESL theories, various kinematic models which use zig-zag and non-polynomial functions have been proposed in the literature. Many contributions on the subject are

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given in publications by Carrera [10,17], Carrera et al. [18], Mantari et al. [19–21], Filippi et al. [22,23], Tornabene et al. [24,25], Viola et al. [26,27], Nguyen et al. [28] and Sayyad and Ghugal [29]. It is clear that many attempts have been made to develop a reliable ESL model able to capture every aspect of the complex nature of the sandwich material.

In order to identify the origin of the lack of accuracy of a particular theory, different theories need to be properly compared and tested. This is not so straightforward if an axiomatic theory is employed. On the contrary, when using unified formulations, theories with different order of expansions can be directly compared, because no changes in the theory or solution method are performed to carry out the comparison. A unified formulation offers the advantage of providing a systematic approach for developing formulations based on ESL and LW in the context of the same framework. Since Carrera introduced Unified Formulations in Ref. [11], a vast amount of literature is available regarding Carrera's Unified Formulation (CUF) [12,22,23,30–41]. An interesting extension to CUF is due to Demasi [42–46] in his so-called Generalized Unified Formulation (GUF), where different order can be introduced for the approximation of different field variables. D'Ottavio proposed a very flexible variable kinematics modeling technique for composite structures in Ref. [47]. The subdivision of the laminate into sublaminates in conjunction with GUF lead to the so-called Sublaminate-GUF (S-GUF) approach. Benchmark results were derived from the Navier solutions of the strong-form governing equations. D'Ottavio et al. [48] extended [47] to the Ritz method as a solution technique in order to analyze different combinations of boundary conditions and eliminate the restrictions with regard to the stacking sequences. Tornabene et al. [49] introduced a new higher-order structural theory to accurately evaluate the natural frequencies of doubly-curved laminated composite shells. The degrees of freedom of the problem are not defined on the shell middle surface as in a typical ESL model, but they are related to specific points placed along the thickness of the structure according to what is typically done for each layer by the LW theory. The term Equivalent Layer-Wise is introduced to define this approach. Caliri et al. [50] presented stress profiles of thick laminated plates and sandwich structures calculated via Caliri's Generalized Formulation (CGF), which consists on a finite element unified formulation with C-1 continuity of the transverse displacement field.

The refined models employed in this paper are based on Carrera's Unified Formulation. According to CUF, the governing equations are given regarding the so-called fundamental nuclei whose form does not depend on either the expansion order nor on the choices made for the base functions. This important feature allows to analyze any number of kinematic models in a single formulation and software. ESL and LW models were successfully developed in CUF, as reported in Ref. [11]. More details on CUF can be found in Refs. [30,31]. To developed accurate refined theories with lower computational effort, Carrera and Petrolo [32,33] introduced the Axiomatic/Asymptotic Method (AAM). This method consists of discarding all terms that do not contribute to the plate response analysis once a reference solution is defined. This leads to the development of reduced models whose accuracies are equivalent to those of full higher-order models. Examples of axiomatic/asymptotic analyses can be found in Refs. [32–38].

The AAM method was adopted to build the Best Theory Diagram (BTD) by Carrera et al. [39]. The BTD allows one to determine the minimum number of expansion terms – i.e. unknown variables – required to meet a given accuracy; or, conversely, the best accuracy provided by a given amount of variables. To construct BTDs with a lower computational cost, a genetic algorithm was employed by Carrera and Miglioretti [40]. Petrolo et al. [41] presented BTDs for ESL and LW composite plate models based on Maclaurin and Legendre polynomial expansions of the unknown variables

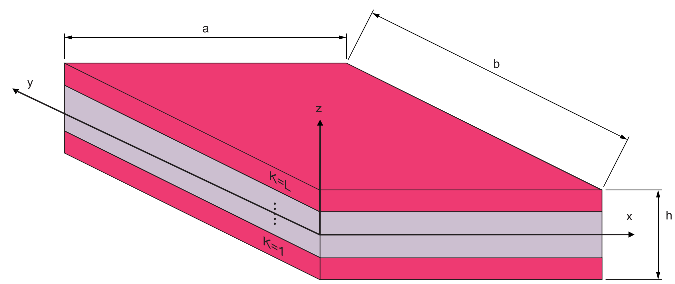


**Fig. 1.** Plate geometry and reference system.

along the thickness. The BTDs reported were build considering a single output parameter, in that case, the stress $\sigma_{xx}$.

The present work employs a new method to develop BTDs considering multiple output parameters – displacements ($u_x$, $u_y$, $u_z$) and stresses ($\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$, $\tau_{xy}$, $\tau_{xz}$, $\tau_{yz}$) – which are evaluated simultaneously in order to obtain similar precision for the output parameters considered. As a result, refined plate models with a control degree of accuracy and computational cost are develop. The plate model employed to develop the BTDs for the analysis of sandwich plates consists of Maclaurin, high-order zig-zag, trigonometric, exponential and hyperbolic expansions. The non-polynomial terms are selected according to Filippi et al. [23]. Genetic algorithms are employed to reduce the computational cost related to the definition of the BTDs. The results obtained show that ESL plate models are able to achieve similar accuracy as a LW plate model if adequate expansion functions are implemented. Moreover, the results show that the addition of non-polynomial and higher-order zig-zag terms can improved the accuracy and computational cost of the refined plate models.

The present paper is organized as follows: a description of the adopted formulation is provided in Section 2; the AAM is presented in Section 3; the BTD for multiple output parameters is introduced in Section 4; the results are presented in Section 5, and the conclusions are drawn in Section 6.

## 2. Carrera unified formulation for plates

The geometry and the coordinate system of the multilayered plate of $L$ layers are shown in Fig. 1, where $x$ and $y$ are the in-plane coordinates while $z$ is the thickness coordinate. The integer $k$ denotes the layer number that starts from the plate-bottom.

According to CUF, the displacement field of a plate structure can be written as follows:

$$\begin{cases} u_x(x,y,z) = F_1(z)u_{x_1}(x,y) + F_2(z)u_{x_2}(x,y) + \cdots \\ \quad + F_{N_{\text{exp}}}(z)u_{x_{N_{\text{exp}}}}(x,y), \\ u_y(x,y,z) = F_1(z)u_{y_1}(x,y) + F_2(z)u_{y_2}(x,y) + \cdots \\ \quad + F_{N_{\text{exp}}}(z)u_{y_{N_{\text{exp}}}}(x,y), \\ u_z(x,y,z) = F_1(z)u_{z_1}(x,y) + F_2(z)u_{z_2}(x,y) + \cdots \\ \quad + F_{N_{\text{exp}}}(z)u_{z_{N_{\text{exp}}}}(x,y). \end{cases} \tag{1}$$

In compact form:

$$\boldsymbol{u}(x,y,z) = F_\tau(z) \cdot \boldsymbol{u}_\tau(x,y) \quad \tau = 1, 2, \ldots, N_{\text{exp}} \tag{2}$$

where $\boldsymbol{u}$ is the displacement vector $(u_x, u_y, u_z)$ whose components are the displacements along the $x$, $y$ and $z$ reference axes. $F_\tau$ are the expansion functions and $\boldsymbol{u}_\tau$ $(u_{x_\tau}, u_{y_\tau}, u_{z_\tau})$ are the displacements variables. $N_{\text{exp}}$ is the number of terms of the expansion.

If an ESL scheme is employed, a multilayered heterogeneous plate is analyzed as a single equivalent lamina. In this case, $F_\tau$ functions can be considered as Maclaurin functions of $z$, defined as $F_\tau = z^{\tau-1}$. The number of unknown variables is not dependent



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