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A complete rotor–stator coupling method for frequency domain analysis of turbomachinery unsteady flow

Dingxi Wang *, Xiuquan Huang

School of Power and Energy, Northwestern Polytechnical University, 127 Youyixi Road, Xi'an, Shaanxi, 710072, China

A B S T R A C T

The paper presents a rotor–stator coupling method for analyzing unsteady flow field within multi-blade-row turbomachines using a frequency domain method. The coupling method is called time and space mode decomposition and matching method. It is based upon coordinate transformation and Fourier transformation to extract relevant time and space modes represented by frequency of unsteadiness, nodal diameter and Fourier coefficients. Detailed procedures together with relevant formula are established to identify the matching time and space modes across an interface and calculate corresponding mode coefficients. The mode coefficients are then matched across an interface with the use of a one-dimensional non-reflective implementation. To verify the correctness and effectiveness of the proposed interface treatment, a transonic compressor was used as a test case. The unsteady flow field within it was analyzed using a time domain time marching method to obtain the results as a reference. Then various frequency domain solutions with different resolution of unsteady frequencies were performed. Comparison of amplitude of different static pressure harmonics between frequency domain solutions and the reference time domain solution was made and discussed.

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1. Introduction

Flow field within turbomachines is inherently unsteady due to sources of blade row interaction and/or blade vibration. To assess blading aeroelasticity in terms of forced vibration and flutter, there is a need to analyze the unsteady flow field. This unsteady flow is quite often strongly, if not completely, periodic in time with the frequencies of unsteadiness being known *a priori* (e.g. blade passing frequency, vibration frequency and their multiples). Resolving the unsteady flow in time domain using time marching methods has been too costly in terms of CPU time and memory usage, particularly for routine designs, even with nowadays computing resources. To significantly reduce the time cost of analyzing the unsteady flow, a few frequency domain methods have been proposed to make use of the time periodic feature of the flow field and the cyclically symmetric feature of the structure of turbomachines. These methods include the linear harmonic method [1], the nonlinear harmonic method [2,3], the harmonic balance method [4–7] and the time spectral method [8–10]. All these methods feature Fourier representation of the unsteady flow field and single passage computational domain together with phase shift boundary condition at the geometrically periodic boundaries.

Turbomachines are quite often designed to have many blade rows with rotors and stators being arranged alternately. For either forced response or flutter analysis, coupling adjacent blade rows in an analysis can make significant difference [11]. To analyze the unsteady flow in a multi-blade-row environment using a frequency domain method, a rotor–stator coupling method is an essential element. Though the earliest effort in applying frequency domain methods to computational domains consisting of multiple blade rows [3] started more than a decade ago, there has been continuous effort up to date in this direction [12–15,11,6,16,17]. The rotor–stator coupling method is vitally important for the application of a frequency domain method to computational domains consisting of multiple blade rows, as it can slow down a solution process or even corrupt the solution if it is not properly formulated.

So far all those rotor–stator coupling methods can be categorized into two groups: one is the sliding plane method based group [18,19,12,6,11,16] and the other is the Fourier transformation based group [3,13,14,17]. The sliding plane method has gained more popularity than the other method, as it is conceptually easier to understand. To use the sliding plane method, apart from searching matching cells/points and interpolating solution, there is a need to reconstruct the time series of flow solution. If the number

of harmonics to be resolved in a domain is small, it is imperative to filter unwanted frequency content, otherwise those unresolved frequency content will defer the convergence, and even corrupt the solution leading to solution instability. For the filter operation, it is also necessary to reconstruct and interpolate solution at sufficient number of matching points/time instants, leading to extra time cost. If there are more than one fundamental frequency in a domain and their greatest common divisor is 1 (e.g. in a flutter analysis in a multi-blade-row environment), then the beating period can be very big and the required over-sampling can be prohibitive.

What one can find about the Fourier transformation based coupling method from the open literature is that it involves two aspects: first it performs Fourier transformation to get the relevant Fourier coefficients, second those Fourier coefficients are matched across an interface [13]. Apparently the above information is far from being sufficient for any one interested to implement this approach. From an implementation point of view, one needs a much more detailed procedure addressing the following points:

1. What spatial/temporal modes to be matched across an interface, and how to determine them
2. What are the formula for spatial and temporal Fourier transformation to extract the spatial/temporal modes
3. How to achieve the matching of those Fourier coefficients in a solution process

For a complete Fourier transformation based coupling method, it also needs to provide theoretical basis for the above procedure. This paper aims to present a complete, concise and generic rotor–stator coupling method based upon coordinate transformation and Fourier transformation together with theoretical derivation of relevant formula.

2. Flow governing equations and harmonic balance method

The unsteady Reynolds averaged Navier–Stokes equations together with the Spalart–Allmaras turbulence model [20] in a cylindrical coordinate system has been used in the investigation. The equation system is not repeated here to save space and interested readers can refer to reference [21]. Solution of the equation system is obtained by using a cell-centered finite volume approach. Spatial discretization of the equation is achieved by the well known JST scheme [22] together with eigenvalue scaled numerical dissipation [23]. The time integration in pseudo time is achieved by a five-stage Runge–Kutta method together with the LU-SGS method [24] being used as a residual smoother. The time integration in physical time can be achieved either by the dual time stepping method or by a nonlinear frequency domain method. For the nonlinear frequency domain method, solution instability due to time spectral source term stiffness is dealt with by implicitly accommodating the source term using block Jacobi method together with the LU-SGS method [21]. A V-cycle multi-grid and local time step are also implemented in the code to speed up solution convergence.

In this investigation, the frequency domain method being used is the time spectral form harmonic balance method [4]. It is assumed that the time dependent flow variables can be approximated using truncated temporal Fourier series,

$$Q(x, \theta, r, t) = \bar{Q}(x, \theta, r) + \sum_{i=1}^{n} \left[Q_{A,i}(x, \theta, r) \sin\omega_i t + Q_{B,i}(x, \theta, r) \cos\omega_i t \right] \quad (1)$$

where n is the number of frequencies being retained in an analysis, $\bar{\ }$ denotes the time averaged quantity, $Q_{A,i}$ and $Q_{B,i}$ denote the Fourier coefficients for unsteadiness corresponding to the ith fre-

quency with the angular frequency of ω_i. $\bar{Q}$, $Q_{A,i}$ and $Q_{B,i}$ are all functions of spatial coordinates only, and independent of time.

With Equation (1), the time derivative in an unsteady governing equation can be approximated by

$$\frac{\partial Q}{\partial t} = \sum_{i=1}^{n} \left(Q_{A,i} \cos\omega_i t - Q_{B,i} \sin\omega_i t \right) \omega_i \quad (2)$$

This provides the chance to convert this time derivative term to a source term, transforming the original unsteady equation system to a quasi steady equation system.

There are $2n+1$ sets of unknowns for n harmonics on the right hand side of Equation (1). Hence at least $2n+1$ sets of equations are required to obtain the solutions. This is achieved by solving the equations at $2n+1$ time instants. The solution vector at $2n+1$ time instants (Q) is related to the Fourier coefficients Q^* through the Fourier transform matrix.

$$Q = D Q^* \quad (3)$$

where

$$Q = \begin{bmatrix} Q(t_1) \\ Q(t_2) \\ \dots \\ Q(t_{2n+1}) \end{bmatrix} \quad (4)$$

$$D = \begin{pmatrix} 1 & \sin\omega_1 t_1 & \cos\omega_1 t_1 & \dots & \sin\omega_n t_1 & \cos\omega_n t_1 \\ 1 & \sin\omega_1 t_2 & \cos\omega_1 t_2 & \dots & \sin\omega_n t_2 & \cos\omega_n t_2 \\ \dots \\ 1 & \sin\omega_1 t_{2n+1} & \cos\omega_1 t_{2n+1} & \dots & \sin\omega_n t_{2n+1} & \cos\omega_n t_{2n+1} \end{pmatrix} \quad (5)$$

$$Q^* = \begin{bmatrix} \bar{Q} \\ Q_{A,1} \\ Q_{B,1} \\ \dots \\ Q_{A,n} \\ Q_{B,n} \end{bmatrix} \quad (6)$$

Q^* can be obtained from Q via the inverse transform of Equation (3),

$$Q^* = D^{-1} Q \quad (7)$$

With Equation (3) and Equation (7), Equation (2) can be denoted as follows

$$\frac{\partial Q}{\partial t} = D_t Q^* = D_t D^{-1} Q = E Q \quad (8)$$

$$D_t = \begin{pmatrix} 0 & \omega_1 \cos\omega_1 t_1 & -\omega_1 \sin\omega_1 t_1 & \dots & \omega_n \cos\omega_n t_1 & -\omega_n \sin\omega_n t_1 \\ 0 & \omega_1 \cos\omega_1 t_2 & -\omega_1 \sin\omega_1 t_2 & \dots & \omega_n \cos\omega_n t_2 & -\omega_n \sin\omega_n t_2 \\ \dots \\ 0 & \omega_1 \cos\omega_1 t_{2n+1} & -\omega_1 \sin\omega_1 t_{2n+1} & \dots & \omega_n \cos\omega_n t_{2n+1} & -\omega_n \sin\omega_n t_{2n+1} \end{pmatrix} \quad (9)$$

For implementation, D is calculated beforehand and inversed numerically before it is multiplied with D_t to get the matrix E. The matrix E is therefore stored for use in a solution process to evaluate the time spectral source term.

For situations with one fundamental frequency, the $2n+1$ time instants are chosen to be equally spaced over the time period corresponding to the fundamental frequency. For situations with more than one fundamental frequency, extra care is needed when time instants are chosen [6,25]. The principle is that the time instants are chosen in such a way that the condition number of the Fourier transform matrix D is minimized. This can be achieved either by optimization (this is done once at the start of a solution) or the Gram–Schmidt process together with over-sampling.