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Trajectory optimization for solar sail in cislunar navigation constellation with minimal lightness number

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ARTICLE INFO

Article history:
Received 13 December 2016
Received in revised form 19 July 2017
Accepted 28 August 2017
Available online xxxx

Keywords:
Cislunar navigation
Trajectory optimization
Hamiltonian-structure-preserving controller
Differential evolution algorithm

ABSTRACT

In view of the limitations of the existing libration-point satellite navigation systems in cislunar space, this paper replaces satellites with solar sails to construct a cislunar navigation constellation and is devoted to the trajectory optimization of solar sails to minimize the lightness number control. The Artificial Lagrangian Points (ALPs) yielded by solar sail in the Sun–Earth+Moon system benefit from the advantages of numberless equilibria and out-of-plane displacement, when compared with the classical Lagrangian points. Limited to the manufacturing of sail film in practice, the candidate constellation architecture in the shape of a cube is constructed based on the optimization of the average lightness number required at ALPs. Considering the lunar gravity, the Hamiltonian-structure-preserving (HSP) controller achieved by changing the sail's attitude and lightness number is developed to stabilize the sails' trajectories near the ALPs. Moreover, an optimal quasi-periodic trajectory with minimum lightness number control is searched for through differential evolution algorithm evolving the controller gains and initial states of orbits. There are three important contributions of the trajectory optimization for a sail in the cislunar navigation constellation: firstly, the large amounts of ALPs break the restrictions on the number and plane of the five classical Lagrangian equilibrium solutions to enlarge the selection of constellations; secondly, the station keeping tool HSP controller powerfully ensures the boundedness of the ALP's trajectory; thirdly, using the optimization algorithm to generate ALP orbits effectively avoids the time consumption of differential correction, which is more convenient and general for the natural trajectory design of ALPs.

1. Introduction

As the only natural satellite of the Earth, the Moon serves as the starting point for deep space exploration activities. Russia intends to establish a permanent base on the Moon before 2030; China [1] and United States [2] both identified the observation and survey of the lunar far side as top priorities in their decadal disciplinary development strategy in space science. It could be expected that the data and material exchange between the Earth and Moon will experience the development peak in the next few years. However, limitations of the ground-based navigation antennas, and the defects present in other infrastructure, such as the orbit determination delay and ground management pressure in the flight control system of the spacecraft-ground loop, could likely cause future large-scale round trips between the Earth and Moon to fail. Thus, if a cislunar navigation system similar to GPS can be constructed, it will make the autonomous flight possible and trigger a positive cycle of the cislunar exchange and communication.

Motivated by this, the study of cislunar navigation systems has created great worldwide interest among the astronautic organizations. Benefiting from the special location and mechanical properties, libration points in the circular restricted three-body problem (CR3BP) [3], i.e., the classical Lagrangian points L_1, L_2, L_3, L_4 and L_5, have been considered as an important method and possible solution to the construction of the cislunar navigation system. The concept of libration-point satellites for lunar communications was first proposed by Farquhar [4], who used only two satellites—one stationed at the interior libration point L_1, and the other following a trajectory about the exterior libration point L_2 to maintain line-of-sight contact. After that, a diverse range of the Earth–Moon libration-point satellite constellations have been designed such as: a triangle configuration composed of L_3, L_4 and L_5 [5], a constellation of four satellites displaced over L_3, L_4, L_5 and L_2-Halo orbit [6], an Earth–Moon L_2, L_4, L_5 three-satellite constellation and an L_1, L_2, L_4, L_5 four-satellite constellation [7]. These researches above reveal the superiority and potential of libration points' ap-

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http://dx.doi.org/10.1016/j.ast.2017.08.042

Please cite this article in press as: X. Pan et al., Trajectory optimization for solar sail in cislunar navigation constellation with minimal lightness number, Aerosp. Sci. Technol. (2017), http://dx.doi.org/10.1016/j.ast.2017.08.042

plications in cislunar navigation to a certain degree. However, there are also two primary limitations of the existing Earth–Moon libration-point navigation architectures: firstly, the number of classical Lagrangian points (CLPs) in the cislunar space is limited to five. Even though the satellites can be placed on the Halo orbits nearby, the range of the motion is restricted; secondly, all the satellites in navigation systems stationed at the Earth–Moon CLPs are confined to the same plane, which weakens the navigation performance to some extent.

Inspired by the ground-based global navigation satellite systems (e.g. GPS, GLONASS, BeiDou, etc.) [8], this paper considers the cislunar navigation constellation composed by displaced solar sails, in which artificial Lagrangian points of the sail-CR3BP are yielded to construct the solar sail constellation dispersedly. Influenced by the gravity from the Moon, solar sails can neither be stationary at artificial Lagrangian points, nor move in bounded orbits nearby, thus a controller achieved by changing the sail's lightness number and attitude is implemented to stabilize the sails' motion. However, due to the complicated deployment and folding process of solar sail membrane, the accessible variation of sail's lightness number in the mechanism is quite small. In the solar sail's restricted N-body problem, the natural periodic or quasi-periodic orbit is the one generated with the minimum change of lightness number, instead of the special solution of CR3BP as a halo orbit [9,10]. For instance, Waters and McInnes [11] obtained the natural trajectories with zero change of lightness number in the Sun–Earth–sail CR3BP. However, under the extra lunar gravitation considered in this paper, there could be no periodic or quasi-periodic trajectory generated without lightness number changing. Based on this fact, this paper is devoted to the trajectory optimization for solar sail in cislunar navigation constellation with the minimal lightness number control.

The remainder of this paper is organized as follows: In Section 2, the dynamical model is established in the Sun–Earth+Moon rotating reference frame, then the artificial Lagrangian points of the sail-CR3BP are yielded and demonstrated. Section 3 constructs a simple candidate constellation architecture to select eight artificial Lagrangian points for sail's station keeping based on the minimum average lightness number required. Section 4 rewrites the dynamical model of solar sail by adding the lunar gravity and control force to design the bounded nominal trajectories of navigated sails. The powerful tool Hamiltonian-structure-preserving controller achieved by changing the sail's attitude and lightness number is developed to guarantee the boundedness of sails' trajectory. Then the parameters of controller and initial states of trajectory are optimized by the differential evolution algorithm to minimize the lightness number control required. The final search results of optimization algorithm are demonstrated, which show that all the lightness number's variation for the sails' trajectories design in navigation constellation can be easily accepted in the mechanism.

2. Sail constellation around the artificial Lagrangian points

2.1. Artificial Lagrangian points by solar sail

In the circular restricted three-body problem, there are five well-known equilibrium solutions: L_1, L_2, L_3, L_4 and L_5 [3]. Since the radiation pressure exerted on the infinitesimal mass has been considered in the restricted three-body problem, new additional equilibria are generated by solar sail for a wider application. Particularly, for the sail-CR3BP, solar sail can regulate the direction and magnitude of solar radiation pressure force by orienting the attitude and changing the lightness number respectively. Therefore, a continuum of new equilibria parameterized by the solar sail lightness number and attitude can be generated, named the artificial Lagrangian points (ALPs).

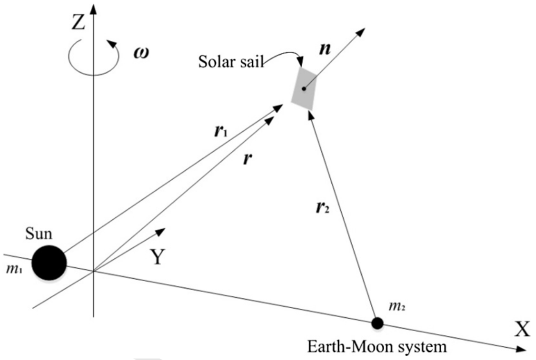

Fig. 1. Solar sail circular restricted three-body problem.

In this paper, the ideal, perfectly reflecting solar sail is considered, which means the sail is assumed regardless of the optical degradation and deformation. Except for the gravitational forces and the solar radiation pressure force, many disturbing forces, such as force caused by the solar wind and the finiteness of the solar disk are neglected for simplification [12].

The bi-circular model is established where the Sun, the Earth–Moon system and the sail consist of the CR3BP. Supposing that the Sun and Earth are revolving in circular orbits around their center of mass and the Moon is moving in a circular orbit around the Earth. To simplify the equations, the units of time, length, and mass are chosen so that the angular velocity of rotation of the Sun and the Earth–Moon system around their barycenter, the distance between the primary masses, the sum of the two primary masses, and the gravitational constant are all taken to be scaled. With these units normalized, the mass of the Earth–Moon system is defined as μ, then the mass of the Sun is $1-\mu$. The Sun–Earth+Moon rotating reference frame is established as shown in Fig. 1, where m_1 denotes the Sun, and m_2 denotes the Earth–Moon system. The origin is taken at the barycenter of the Sun–Earth+Moon system, the X-axis points along the line from the Sun to the barycenter of the Earth–Moon system, the Z-axis is the axis of rotation, and the Y-axis follows the right-handed rule. It is noted that the two primary masses in this rotating reference frame have fixed positions while the Moon is rotating around the Earth. The solar sail position vectors are defined with respect to the origin as $\boldsymbol{r} = (x, y, z)^T$, to the Sun as $\boldsymbol{r}_1 = (x+\mu, y, z)^T$, to the Earth–Moon barycenter as $\boldsymbol{r}_2 = (x-1+\mu, y, z)$, to the Earth as $\boldsymbol{r_e} = (x-x_e, y-y_e, z-z_e)$ and to the Moon as $\boldsymbol{r_m} = (x-x_m, y-y_m, z-z_m)$ where (x_e, y_e, z_e) and (x_m, y_m, z_m) are respectively the coordinates of the Earth and the Moon.

Then the dynamics of solar sail in rotating reference frame can be written as

$$\ddot{\boldsymbol{r}} + 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \nabla U = \boldsymbol{a} \tag{1}$$

where $\boldsymbol{\omega}$ is the normalized angular velocity of rotation, $\boldsymbol{a}$ is the solar radiation pressure acceleration and U is the three-body potential function, defined by McInnes [13] as:

$$U = -\left[\frac{1}{2}\left(x^2+y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}\right] \tag{2}$$

$$\boldsymbol{a} = \beta \frac{1-\mu}{r_1^2}(\hat{\boldsymbol{r}}_1 \cdot \boldsymbol{n})^2 \boldsymbol{n}, \tag{3}$$

where $\boldsymbol{n}$ is the normal vector of solar sail deviated from the sun-line and β is the lightness number parameter, defined as the ratio of the solar radiation pressure acceleration to the solar gravitational acceleration.

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